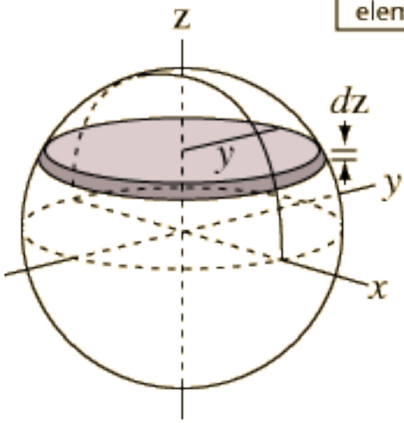
# Solution of the problem from previous training

# Calculating Moment of Inertia




Infinitesmal moment of inertia element

Moment of a thin disk of mass dm

Substituting area times height of disk for volume dV

$$dI = \frac{1}{2}y^2\,dm = \frac{1}{2}y^2\rho\,dV = \frac{1}{2}y^2\rho\pi y^2\,dz$$

Expressing mass dm in terms of density and volume.

Integrating the moment of inertia element from the bottom to the top of the sphere

$$I = \frac{1}{2}\rho\pi\int_{-R}^{R} y^4\,dz = \frac{1}{2}\rho\pi\int_{-R}^{R}(R^2 - z^2)^2\,dz = \frac{8}{15}\rho\pi R^5$$

Radius = $R$

Mass = $M$

Density $= \rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$

$$I = \frac{8}{15}\left[\frac{M}{\frac{4}{3}\pi R^3}\right]\pi R^5 = \frac{2}{5}MR^2$$

http://hyperphysics.phy-astr.gsu.edu/hbase/isph.html#sph3

# Calculating Moment of Inertia (smart and general way to do that)

For spherical planet and arbitrary radial-dependent density

$$I_z = \iiint_V \rho(r) \cdot (x^2 + y^2) dV$$

$$r^2 = x^2 + y^2 + z^2$$

$$I = \int_0^R \rho(r) \cdot f(r) dr$$

You must come to:

$$I = (8\pi / 3) \int_0^R \rho(r) \cdot r^4 dr$$

Use an idea that for the spherically symmetric Earth moments of inertia for rotation around Z, Y and X axis are equal!

# Calculating Moment of Inertia (smart and general way to do that)

$$I_z = \iiint_V \rho \cdot (x^2 + y^2)\,dV$$

$$I_y = \iiint_V \rho \cdot (x^2 + z^2)\,dV$$

$$I_x = \ldots$$

$$r^2 = x^2 + y^2 + z^2$$

$$dV = 4\pi r^2 dr$$

For spherical planet and arbitrary radial-dependent density

$$I_z = I_y = I_x = I$$

# Calculating Moment of Inertia (smart and general way to do that)

$$I_z = \iiint_V \rho \cdot (x^2 + y^2) dV$$

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$$I_x = \iiint_V \rho \cdot (z^2 + y^2) dV$$

$$I_x + I_y + I_z = \iiint_V \rho \cdot 2(x^2 + y^2 + z^2) dV = 2\iiint_V \rho \cdot r^2 dV$$

For spherical planet and arbitrary radial-dependent density

$$I_z = I_y = I_x = I$$

$$3I = 2\iiint_V \rho(r) \cdot r^2 dV = 2\int_0^R \rho(r) \cdot r^2 4\pi r^2 dr = 8\pi \int_0^R \rho(r) \cdot r^4 dr$$

$$I = 8/3\pi \int_0^R \rho(r) \cdot r^4 dr$$

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# Calculating Moment of Inertia (smart and general way to do that)

For spherical planet and arbitrary radial-dependent density

$$I = (8\pi / 3)\int_0^R \rho(r) \cdot r^4 dr$$

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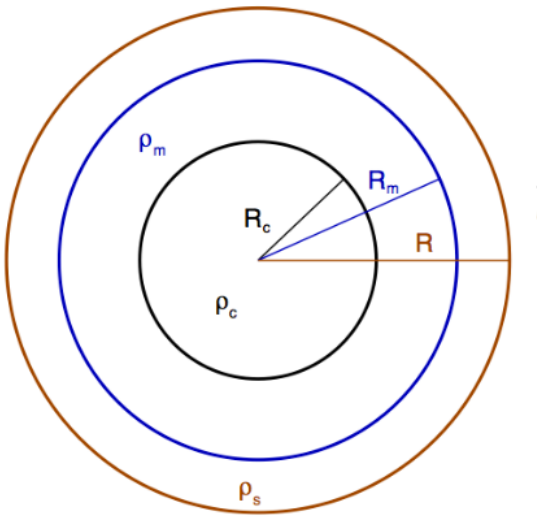


For spherical planet with *n* layers

$$I = (8\pi / 15)(\rho_1 R_1^5 + \rho_2 (R_2^5 - R_1^5) + \ldots + \rho_n (R_n^5 - R_{n-1}^5))$$

$$M = (4\pi / 3)(\rho_1 R_1^3 + \rho_2 (R_2^3 - R_1^3) + \ldots + \rho_n (R_n^3 - R_{n-1}^3))$$