

1. Introduction

Parameterized models of whole mantle convection have been used to study the thermal history of the Earth and to infer some physical properties of the mantle such as radiogenic heat source content.

These models generally show that mantle temperature and heat flow decrease with time, while mantle viscosity increases with time, all in accordance with a cooling Earth. Furthermore, heat flow exceeds radiogenic heat generation at the present day so that the Urey ratio, the ratio of internal heat generation to surface heat flow, is less than unity at present.

An essential feature of these parameterized convection thermal history models is their inclusion of the strong temperature dependence of mantle viscosity.

The thermostat effect of temperature-dependent mantle viscosity regulates the rate of mantle cooling; early in the Earth's thermal evolution, when the mantle is very hot, the viscosity is low, convection is highly vigorous, and the mantle cools rapidly.

The rapid reduction in mantle temperature increases mantle viscosity and reduces the vigor of convection and the rate of mantle cooling. Throughout most of the Earth's thermal history, cooling is gradual, about 100 K/Gyr. Mantle viscosity is not only a strong function of temperature, but it depends sensitively on mantle volatile content as well. Dissolved volatiles in the mantle lower the creep activation enthalpy and thus reduce the viscosity at a given temperature. Thus, a loss of volatiles from the mantle (degassing or outgassing) would stiffen the mantle and require an increase in mantle temperature to maintain a requisite vigor of convection.

However, almost all of the previous parameterized convection thermal history studies have included the dependence of mantle viscosity on volatile content.

The inclusion of a volatile-dependent as well as temperature-dependent mantle viscosity into thermal history modelling is important not only for a correct simulation of mantle evolution, but also for its implications regarding the evolution of the atmosphere.

The dependence of mantle viscosity on both temperature and volatile content produces a strong coupling between mantle thermal evolution and the degassing/regassing history of the mantle (regassing refers to the volatile recharging of the mantle by tectonic processes such as subduction, overthrusting, and delamination).

In the thermal history study of Jackson and Pollack (1987), the degassing history of the mantle was *a priori* specified by assuming explicit functional forms for the dependence of activation enthalpy (or temperature) on time. In the present paper we self-consistently solve for the evolution of both mantle temperature and the mantle degassing/regassing rate.

2. Model Equations

The basic equations of parametrized convection are developed in detail elsewhere; we will only briefly review the key concepts and equations. The kinematic viscosity of the mantle is assumed to have the following temperature dependence:

$$\nu = \bar{\nu} \cdot \exp(A/T) \quad (1)$$

where A is the activation temperature for solid-state creep, T is a characteristic mantle temperature, and $\bar{\nu}$ is a constant. Decaying radiogenetic heat sources are assumed to produce energy in the mantle at a rate Q per unit volume, where:

$$Q = Q_0 \cdot \exp(-\lambda t) \quad (2)$$

and Q_0 and λ are constants, and t is the time.

The Rayleigh number Ra for a convecting mantle is:

$$Ra = \frac{g\alpha \cdot (T - T_s)(R_m - R_c)^3}{\kappa \cdot \nu} \quad (3)$$

where g is the acceleration of gravity, α is the coefficient of thermal expansion, T_s is the surface temperature, R_m and R_c are the mantle outer and inner radii, respectively, and κ is the thermal diffusivity. Mantle heat flow q is parameterized in terms of Ra by [2]:

$$q = \frac{k \cdot (T - T_s)}{R_m - R_c} \cdot \left(\frac{Ra}{Ra_{cr}} \right)^\beta \quad (4)$$

where k is the thermal conductivity, Ra_{cr} is the critical value of Ra for the onset of convection, and β is an empirically determined dimensionless constant found to be around 0.3.

Conservation of energy gives an equation for the time rate of change of the characteristic mantle temperature \dot{T} :

$$\rho c \cdot (R_m^3 - R_c^3) \frac{dT}{dt} = -3R_m^2 q + Q \cdot (R_m^3 - R_c^3) \quad (5)$$

where ρ is the density and c is the specific heat.

Equations (1) to (5) give a first-order non-linear differential equation for the mantle temperature $T(t)$, which, when given the temperature T_0 at time $t = 0$, is easily solved by numerical integration. (We have chosen an adaptable stepsize Runge-Kutta algorithm [13] to integrate (1) to (5)).

This differential equation for $T(t)$ has been the subject of several studies. For example, Schubert et al. [2] and Jackson and Pollack [3] used a constant value for A , and Jackson and Pollack [11] assumed a specific function for the time dependence in A .

3. Activation temperature and degassing/regassing parameterizations

Experiments on the deformation of candidate mantle minerals have revealed that dissolved volatiles such as water weaken the minerals by reducing the activation energy E^* for solid-state creep [14-16]. Activation energy is related to activation temperature by the equation:

$$A = E^* / R \quad (6)$$

where R is the universal gas constant.

Chopra and Paterson [15] give values for the activation energy and weight fraction of water for two samples of “wet” dunite as well as the activation energy for both “dry” samples. The parameterization we use for activation temperature as a function of volatile weight fraction is a simple straight line drawn through the wet and dry points for each type of dunite. Uncertainties in the data and the lack of additional data preclude a more complicated parameterization.

Accordingly, we write:

$$A = \alpha_1 + \alpha_2 \cdot x \quad (7)$$

where x is the volatile weight fraction and α_1, α_2 are constants determined by the straight lines in Fig. 1.

The values of α_1 and α_2 according to Fig. 1 are

$$\begin{aligned} \alpha_1 &= 6.4 \cdot 10^4 K \\ \alpha_2 &= -6.1 \cdot 10^6 K/(\text{weight fraction}) && \text{(Anita Bay dunite)} \\ \alpha_2 &= -8.1 \cdot 10^5 K/(\text{weight fraction}) && \text{(Aheim dunite)} \end{aligned}$$

The smaller value of α_2 is representative of a mantle with a weak dependence of rheology on volatile content; the larger value of α_2 typifies a strong dependence of mantle viscosity on the percentage of volatiles.

Our model requires a prescription for determining the mass of dissolved volatiles in the mantle as a function of time. We have identified the major mechanisms of mantle degassing and regassing and have parameterized them to provide equations for the degassing and regassing rates. The rate of mantle degassing $[\dot{M}_{mv}]_d$ is given by:

$$[\dot{M}_{mv}]_d = \rho_{mv} \cdot d_{melt} \cdot S \quad (8)$$

where ρ_{mv} is the density of volatiles in the mantle, d_{melt} is the average depth from which volatiles are released from the mantle (assuming complete outgassing to this depth), and S is the areal spreading rate for the Earth's mid-ocean ridges. The parameter d_{melt} can be thought of as an "equivalent depth", combining the actual depth of melting with an efficiency factor for the release of volatiles. The rate of regassing $[\dot{M}_{mv}]_r$ is given by

$$[\dot{M}_{mv}]_r = f_{bas} \cdot \rho_{bas} \cdot d_{bas} \cdot S \cdot \chi_r \quad (9)$$

where f_{bas} is the mass fraction of volatiles in the basalt layer, ρ_{bas} is

the density of basalt, d_{bas} is the average thickness of the basalt, and χ_r is an efficiency factor representing the fraction of volatiles that actually enters the deep mantle instead of returning to the surface through back-arc volcanism. The value of d_{bas} can be varied to reflect the added contribution of a subducted sediment layer.

To obtain the spreading rate S as a function of known quantities we use the relation between heat flow q and the average age of subduction of oceanic crust τ [17, eq. 4-128]:

$$q = \frac{2k \cdot (T - T_s)}{\sqrt{\pi \kappa \tau}} \quad (10)$$

where τ is given by

$$\tau = \frac{A_o(t)}{S} \quad (11)$$

and A_o is the area of ocean basins at time t .

The solution of (10) and (11) for S is:

$$S(t) = \frac{q(t)^2 \pi \kappa A_o(t)}{[2k \cdot [T(t) - T_s]]^2} \quad (12)$$

Reymer and Schubert [12] provide a formula for $A_o(t)$ based on the assumption of approximately constant continental freeboard over the last 500 million years:

$$A_o(t) = A_o^* \cdot \left[\frac{V_{oa}^*}{V_o} + \frac{V_{ob}^* \cdot q^*}{V_o \cdot q(t)} \right]^{-1} \quad (13)$$

where V_o is total volume of water in the oceans (assumed constant in time), V_{oa} is the volume of the ocean basins above the peak ridge height, V_{ob} is the volume of the ocean basins below the peak ridge height, and asterisks denote present-day values.

The differential equation for the mass of mantle volatiles is simple:

$$\dot{M}_{mv} = [\dot{M}_{mv}]_r - [\dot{M}_{mv}]_d \quad (14)$$

This equation assumes that no other fluxes of volatiles (such as impact degassing) are significant. Interpretation of (14) requires an assumption about the initial value of M_{mv} . We let:

$$M_{mv}(t = 0) = n_m \cdot M_{ocean} \quad (15)$$

where M_{ocean} is the mass of the Earth's oceans and n_m is the number of ocean masses initially in the mantle. We similarly express the initial value of the mass of volatiles starting out on the surface as:

$$M_s(t = 0) = n_s \cdot M_{ocean} \quad (16)$$

where n_s is the number of ocean masses originally in surface volatile reservoirs.

4. Model parameter values

To provide a starting point from which to begin our study we established a nominal case using the parameter values given in Tables 1 and 2. Table 1 lists the values of the parameters pertaining to the basic convection equations. Table 2 lists the values of the parameters used for the degassing / regassing part of the calculation. Both tables provide references for the assumed parameter values. The value of the depth of melting d_{melt} is derived from an estimate of the depth of the basalt eutectic in the Archean [19, fig. 1]. This value is too large to reflect present conditions, but it is intended to model conditions prevalent in the early history of the Earth when convective vigor was much greater. Since rates of volatile exchange in the model (and presumably in the Earth) are much greater in the early part of a calculation than they are toward the end of a calculation, the value of d_{melt} should represent conditions early in Earth's evolution.

Ringwood [20,21] has estimated that the mass of dissolved water in the mantle is approximately three times that currently in the oceans.

With the assumption that the total amount of water in the mantle-hydrosphere-atmosphere system is conserved, we set $n_m = 4$. The value of Q_0 is iteratively adjusted so that the heat flow q at $t = 4.6$ Gyr is equal to the present day value, $q^* = 70$ mW/m².

Table 3 summarizes how the nominal case parameters were modified to consider other evolutionary scenarios. The nominal case is intended to demonstrate the effects of a strongly volatile-dependent viscosity. Calculation 2 uses a weaker rheological volatile dependence, and case 3 has a volatile-independent rheology.

The first three calculations all resulted in net outgassing from the mantle. Thus for case 4, parameters were adjusted to diminish degassing and promote regassing. Case 5 starts with a highly volatile-charged mantle. Case 6 demonstrates the effects of an initially cool mantle. Computations 7 and 8 test the effects of alternate values for $\bar{\nu}$ and λ used in other parameterized convection models.

5. Parameter values

Table 1

Parameter	Value	Source
$\bar{\nu}$	$2.21 \cdot 10^7 \text{ m}^2/\text{s}$	[11]
λ	$3.4 \cdot 10^{-10} \text{ yr}^{-1}$	[3]
g	9.8 m/s^2	[11]
α	$3 \cdot 10^{-5} \text{ K}^{-1}$	[2,11]
κ	$10^{-6} \text{ m}^2/\text{s}$	[2,11]
R_m	6271 km	[2,11]
R_c	3471 km	[2,11]
T_s	273 K	[2,11]
k	4.2 W/mK	[11]
Ra_{cr}	1100	[2,11]
ρc	$4.2 \cdot 10^6 \text{ J/m}^3\text{K}$	[2,11]
β	0.3	[2,11]

Nominal case convection parameter values

Table 2

Parameter	Value	Source
M_{mantle}	$4.06 \cdot 10^{24}$ kg	[2]
d_m	10^5 m	[19]
f_{bas}	0.03	[12]
d_{bas}	$5 \cdot 10^3$ m	[12]
ρ_{bas}	$2.95 \cdot 10^3$ kg/m ³	[17]
χ_r	0.8	-
M_{ocean}	$1.39 \cdot 10^{21}$ kg	[27]
n_m	4.0	[20,21]
n_s	0	-
A_o^*	$3.1 \cdot 10^{14}$ m ²	[18]
V_{oa}^*	$7.75 \cdot 10^{17}$ m ³	[18]
V_{ob}^*	$3.937 \cdot 10^{17}$ m ³	[18]
V_o	$1.1687 \cdot 10^{18}$ m ³	[18]
q^*	0.07 W/m ²	[17]

Nominal case degassing/regassing parameter values

Table 3

Calculation	Conditions (modifications to the nominal case)
1	nominal
2	Aheim dunite parameterization
3	constant A , $\alpha_1 = 5.60199 \cdot 10^4$ K, $\alpha_2 = 0$
4	$d_{bas} = 6$ km, $d_{melt} = 50$ km, $n_m = 4$, $n_s = 2$
5	$n_m = 10$, $n_s = 0$
6	$T(0) = 2000$ K
7	$\bar{\nu} = 1.65 \cdot 10^2$ m ² /s [2]
8	$\lambda = 4.5 \cdot 10^{-10}$ yr ⁻¹ [3]

Summary of computations

Table 4

Calculation	Q_0 ($\text{J}/\text{m}^3 \text{ s}$)	T (K)	Mantle volatile loss	Urey ratio	Viscosity (m^2/s)	Rayleigh number
1	$1.43093 \cdot 10^{-7}$	2480.0	1.48	0.743	$4.953 \cdot 10^{17}$	$2.876 \cdot 10^7$
2	$1.40006 \cdot 10^{-7}$	2635.3	1.25	0.727	$6.651 \cdot 10^{17}$	$2.297 \cdot 10^7$
3	$1.43508 \cdot 10^{-7}$	2372.1	1.56	0.745	$3.986 \cdot 10^{17}$	$3.398 \cdot 10^7$
4	$1.43540 \cdot 10^{-7}$	2313.0	-0.81	0.745	$3.522 \cdot 10^{17}$	$3.738 \cdot 10^7$
5	$1.45181 \cdot 10^{-7}$	2447.1	7.03	0.754	$4.641 \cdot 10^{17}$	$3.024 \cdot 10^7$
6	$1.46151 \cdot 10^{-7}$	2434.5	0.86	0.759	$4.526 \cdot 10^{17}$	$3.082 \cdot 10^7$
7	$1.63900 \cdot 10^{-7}$	1762.2	1.82	0.851	$9.007 \cdot 10^{16}$	$1.067 \cdot 10^8$
8	$2.15071 \cdot 10^{-7}$	2490.9	1.63	0.673	$5.060 \cdot 10^{17}$	$2.829 \cdot 10^7$

Model results

6. Results

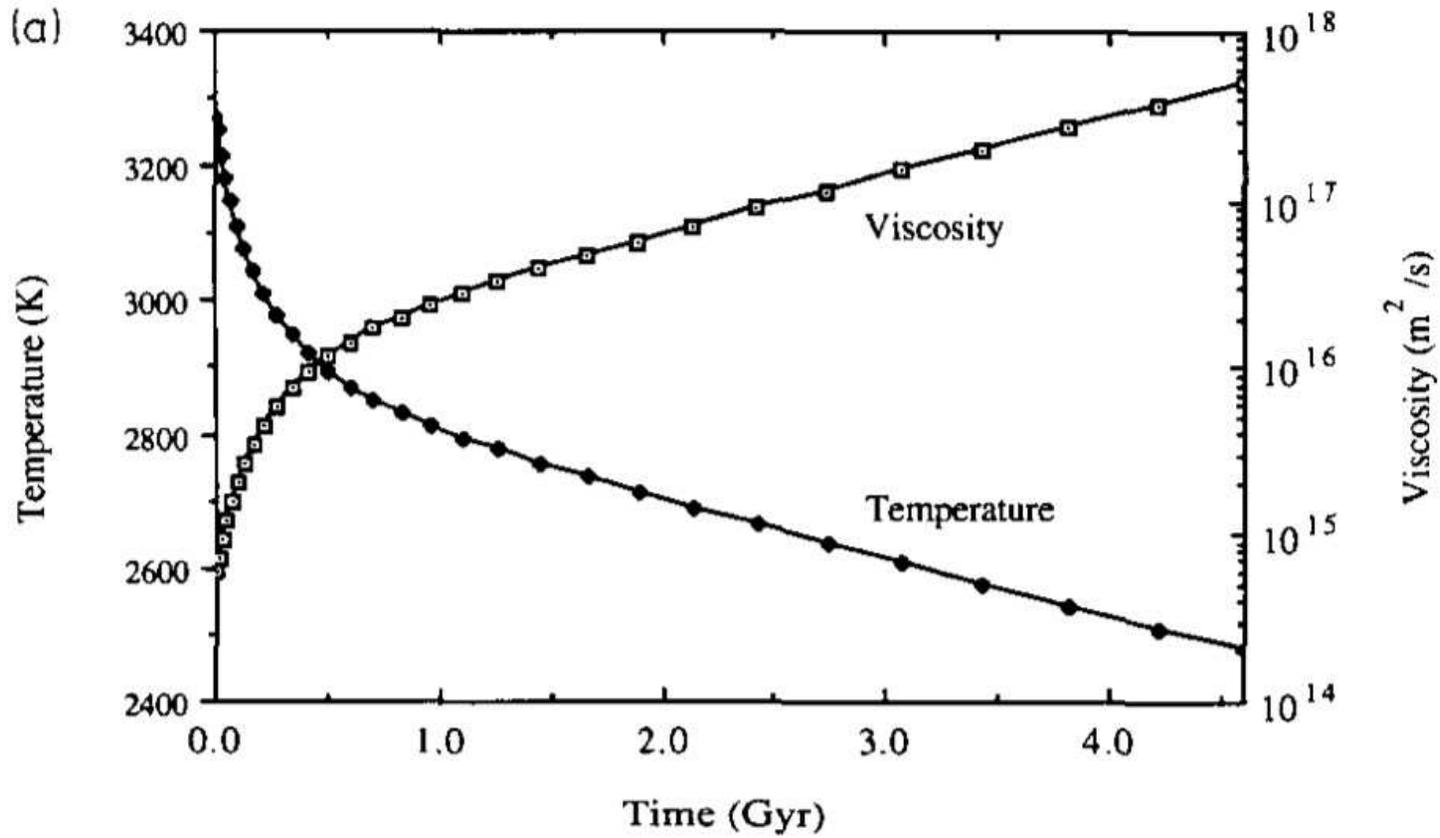


Figure 4a: “Average” mantle temperature T and mantle viscosity ν as functions of time for the nominal case (calculation 1)

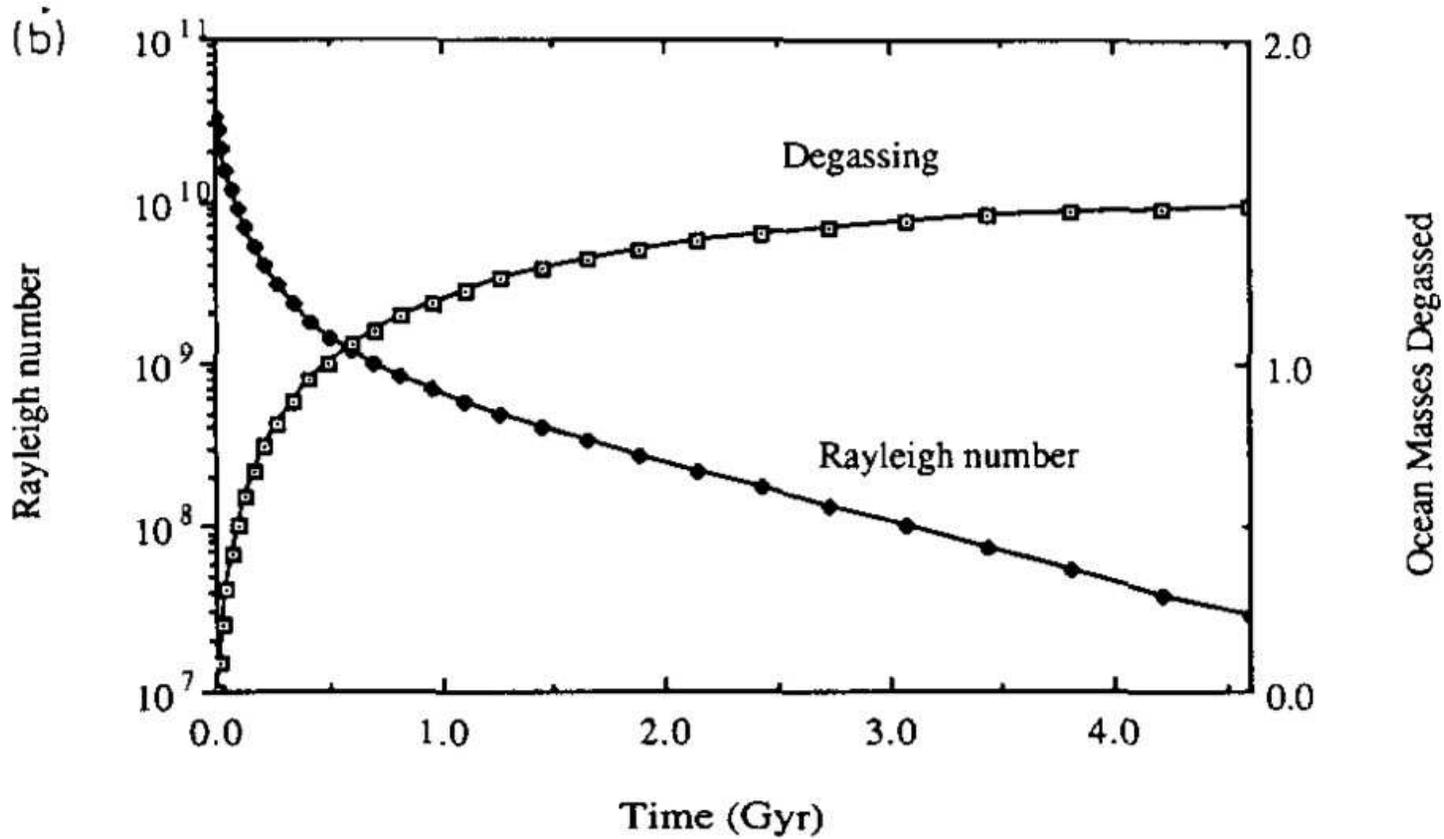


Figure 4b: Mantle Rayleigh number and amount of outgassing from the mantle (in units of ocean masses) as functions of time for the nominal case

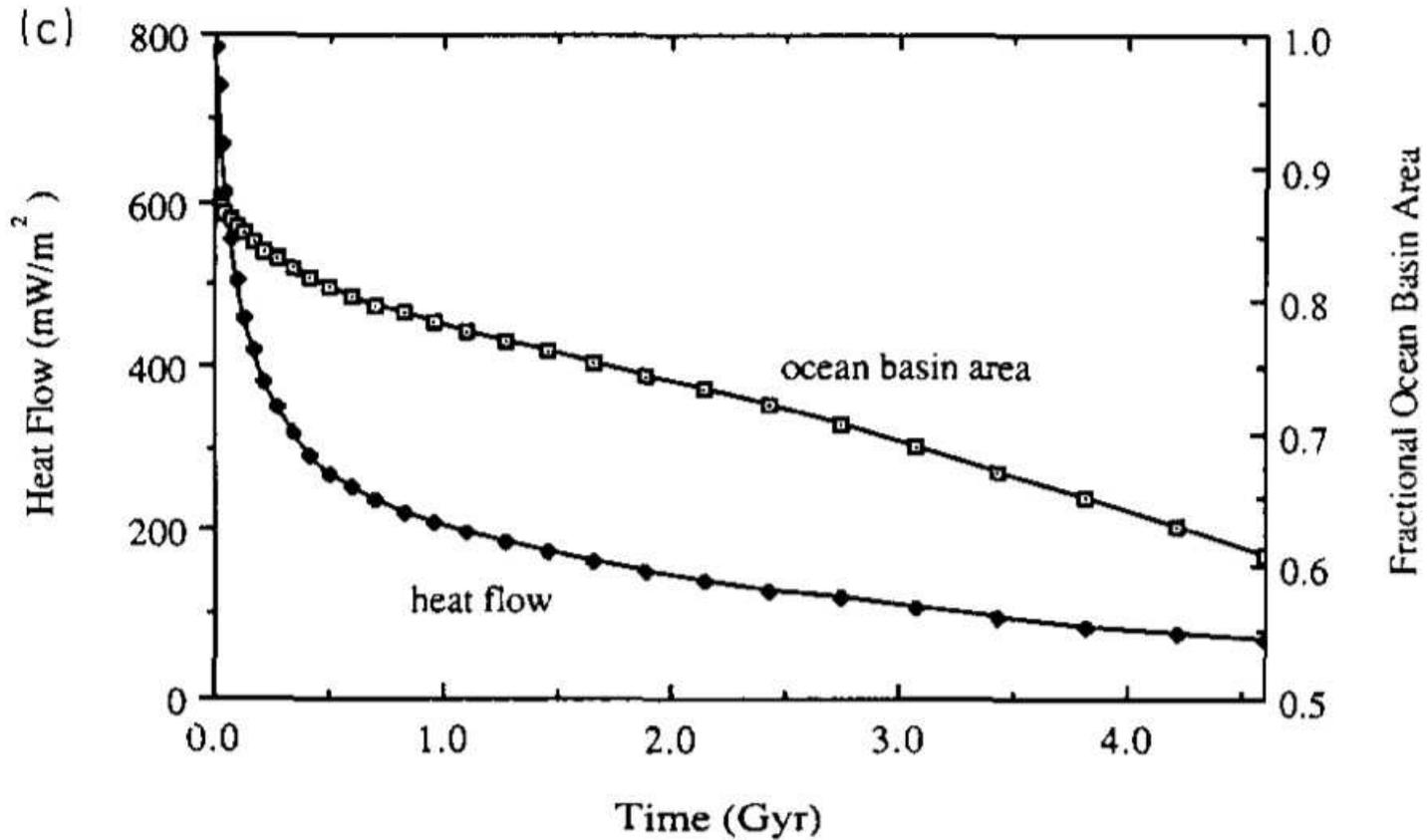


Figure 4c: Heat flow from the mantle and the normalized area of the ocean basins (from the parameterization of the spreading rate S) as functions of time for the nominal case

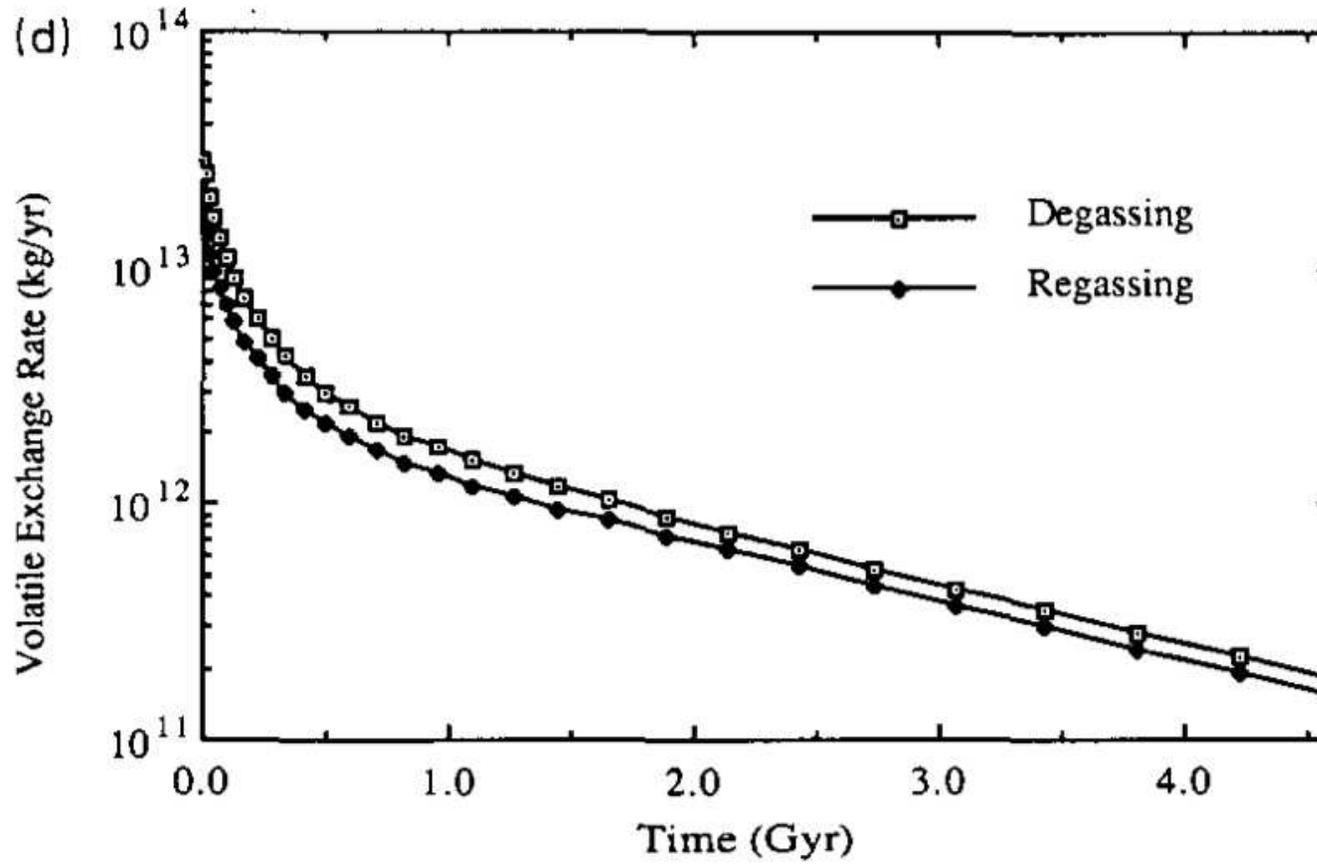


Figure 4d: Mantle degassing and regassing rates as a function of time for the nominal case