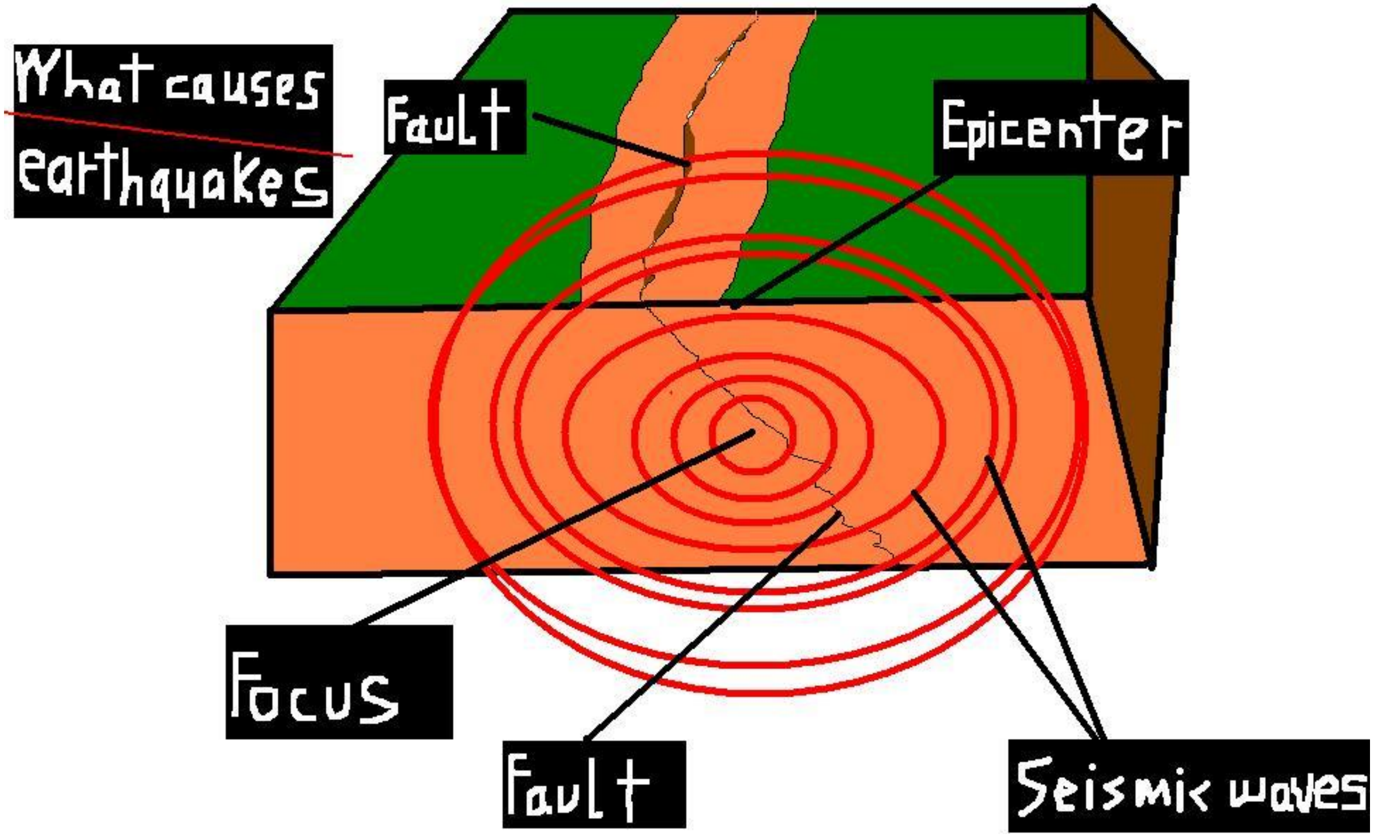
Rate-and-State Friction Law

Iskander Muldashev, Stephan Sobolev
GFZ, Section 2.5 Geodynamic Modeling

Reason for Earthquake



Emaze.com

Earthquake Machine

Earthquake machine
Single-block model

Described by John C. Lahr
USGS Emeritus Seismologist

iris.edu

Rate-and-State Friction

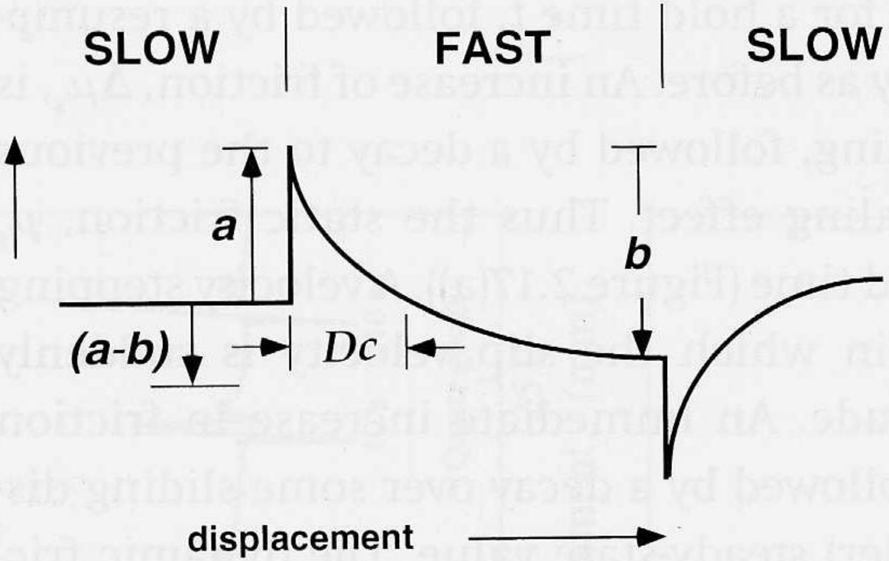


Fig. 2.18. Schematic diagram showing the response to *e*-fold velocity changes and defining the terms in the rate-state friction law.

(Scholz, 1998)

$$\bar{\sigma} = \sigma - pf$$

$$\boldsymbol{\tau = \mu \times \bar{\sigma} = \left[\mu_0 + a \times ln\left(\frac{V}{V_0}\right) + b \times ln\left(\frac{\theta}{\theta_0}\right)\right] \times \bar{\sigma}}$$

$$\boldsymbol{\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}}$$

(Dieterich, 1981)

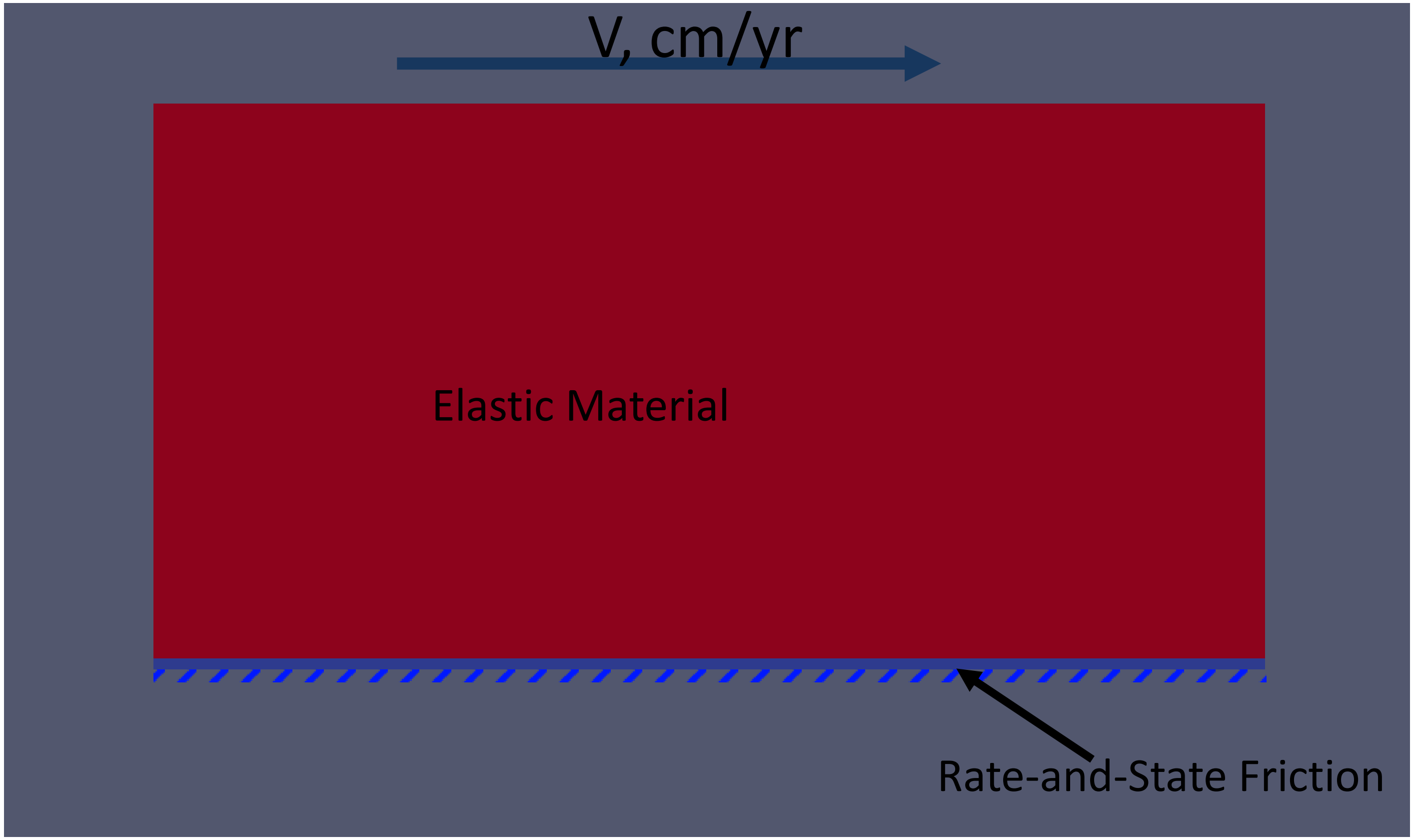
Steady State

$$\boldsymbol{\mu_{ss} = \mu_0 + (a - b) \times ln\left(\frac{V}{V_0}\right)}$$

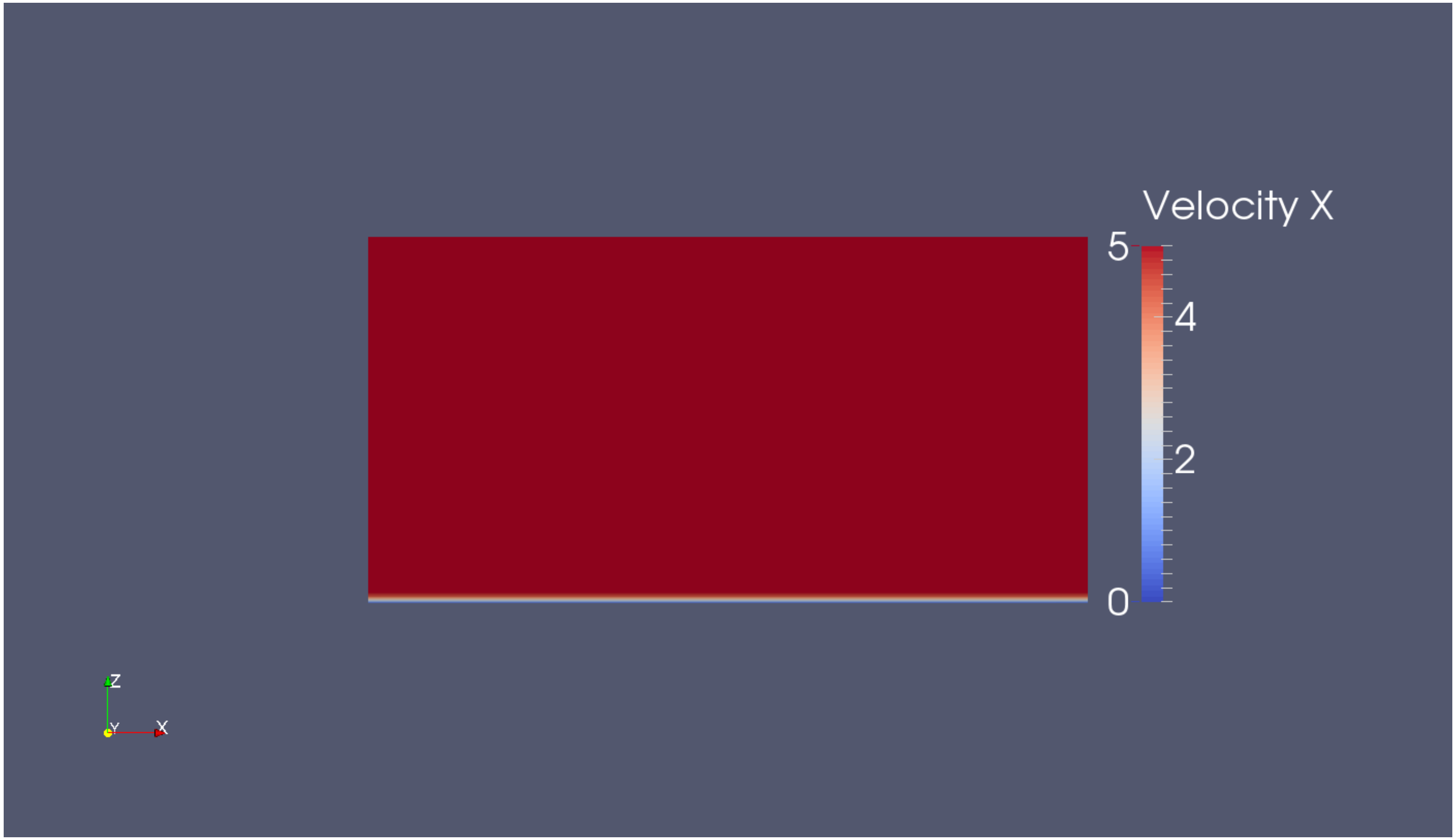
(a-b) > 0 velocity strengthening (stable sliding)

(a-b) < 0 velocity weakening (unstable sliding)

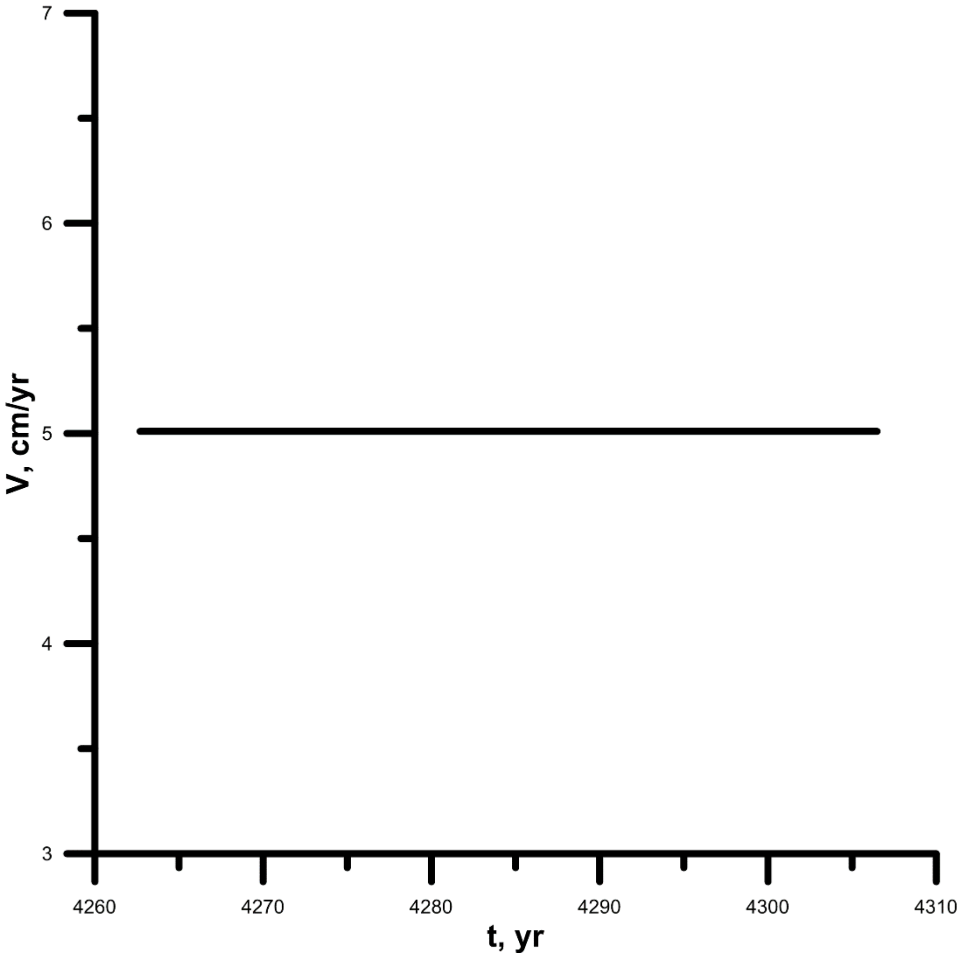
Simple Shear Model



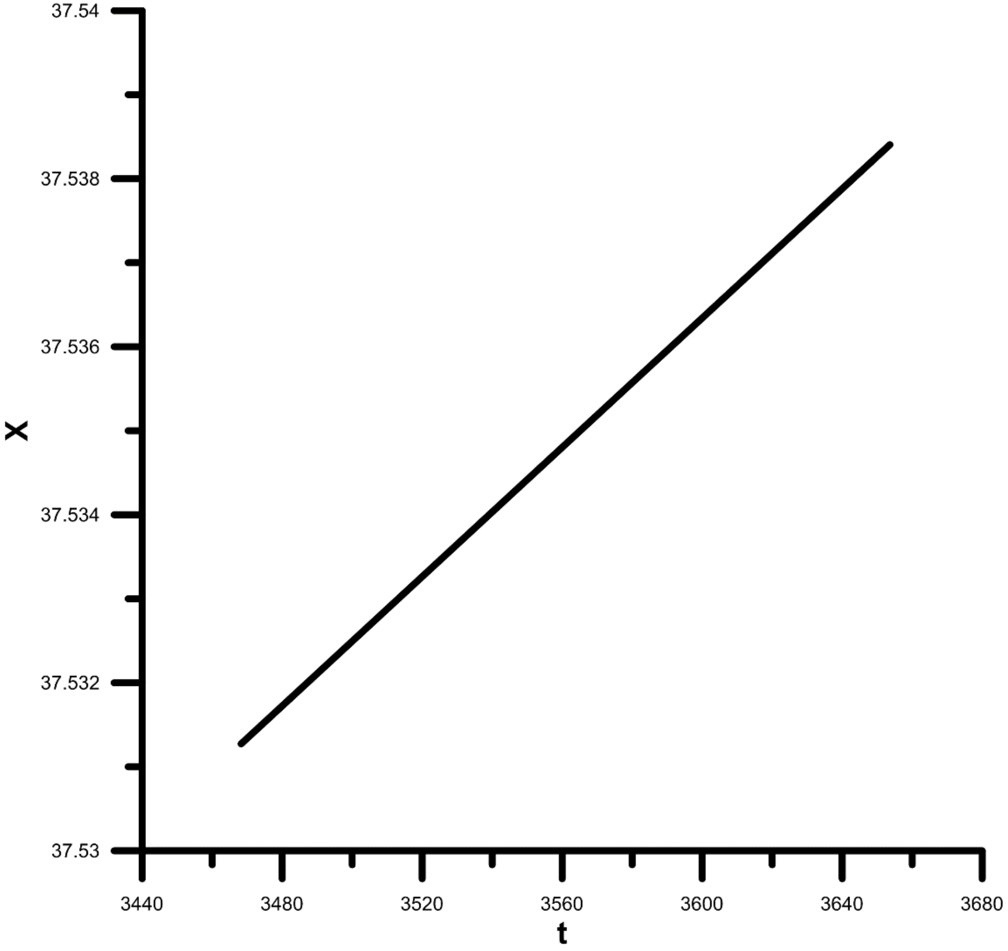
(a-b)>0 strengthening



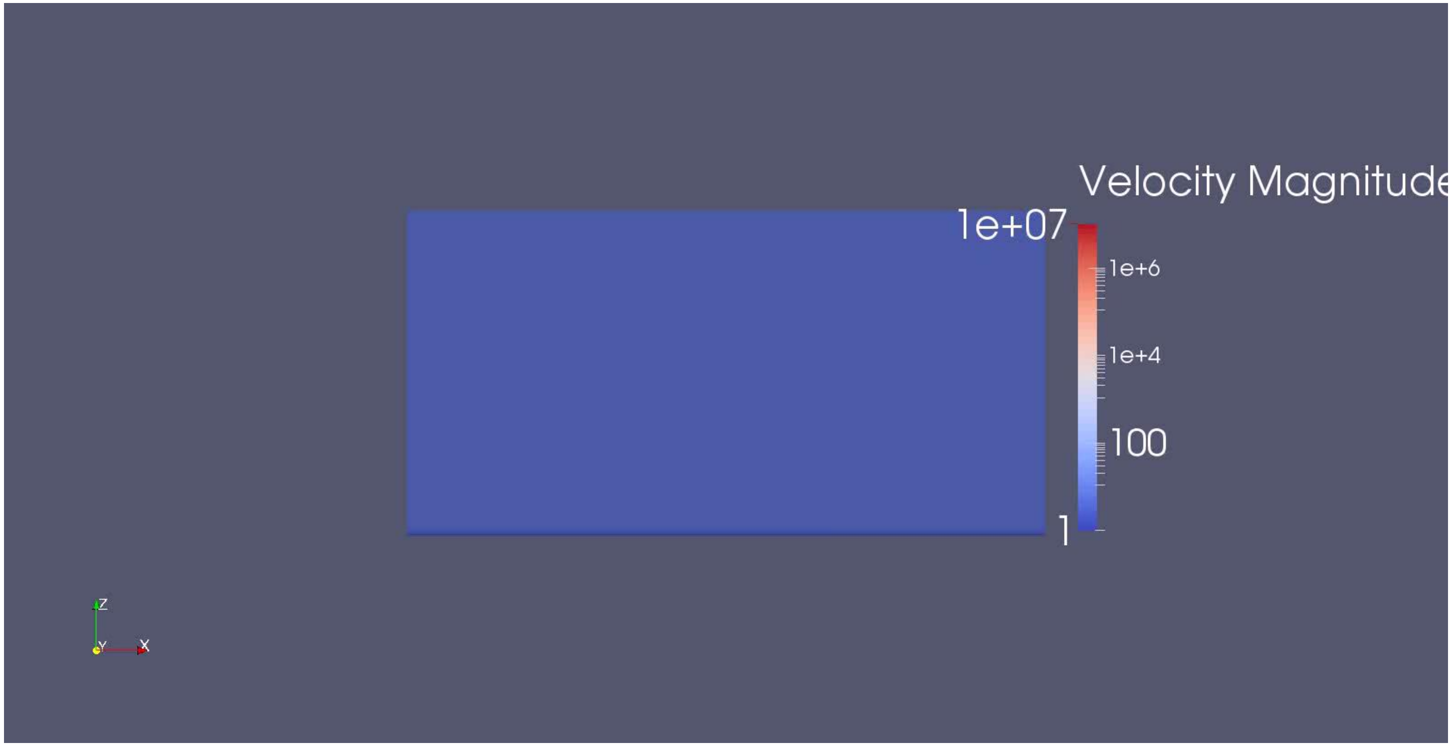
(a-b)>0 (strengthening)



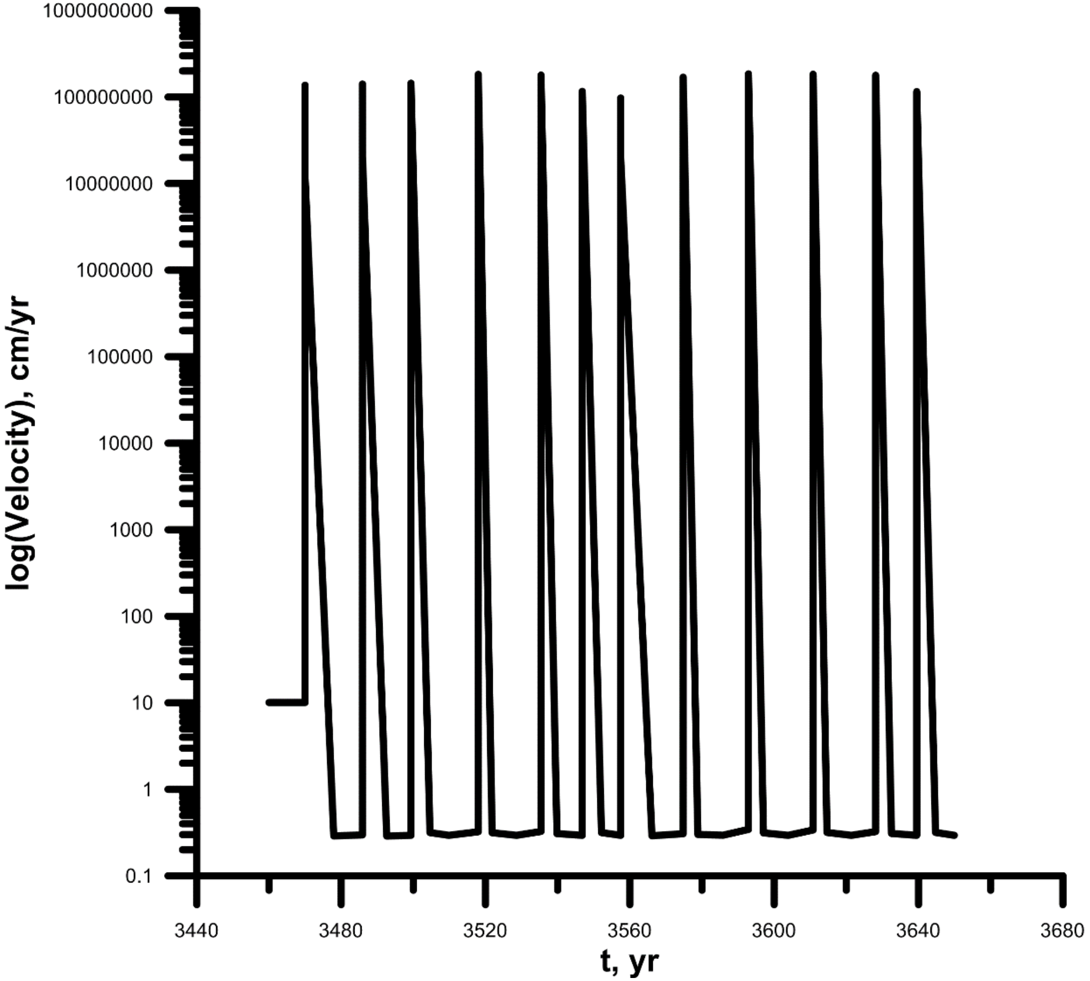
(a-b)>0 (strengthening)



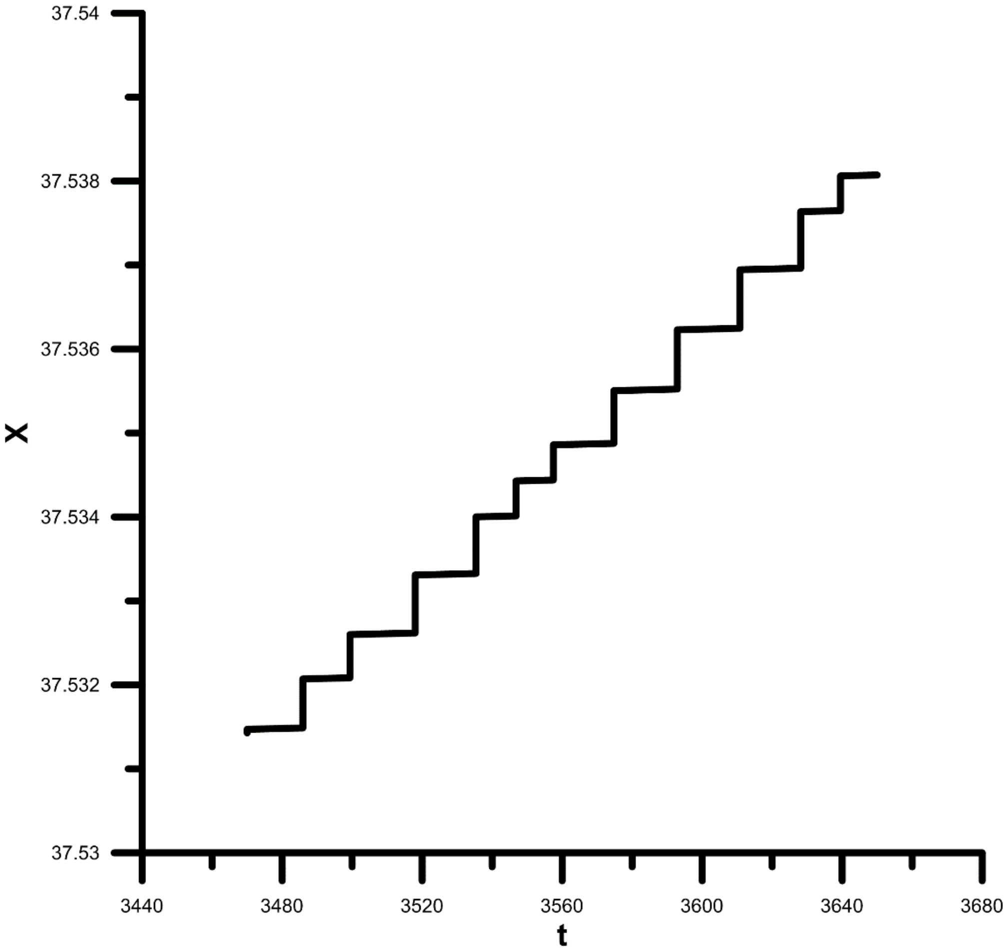
(a-b)<0 (weakening)



(a-b)<0 (weakening)



(a-b)<0 (weakening)



Ruina-Dieterich Equation

$$\tau = \mu \times \bar{\sigma} = \left[\mu_0 + a \times ln\left(\frac{V}{V_0}\right) + b \times ln\left(\frac{\theta}{\theta_0}\right)\right] \times \bar{\sigma}$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}$$

Rate-and-State Friction

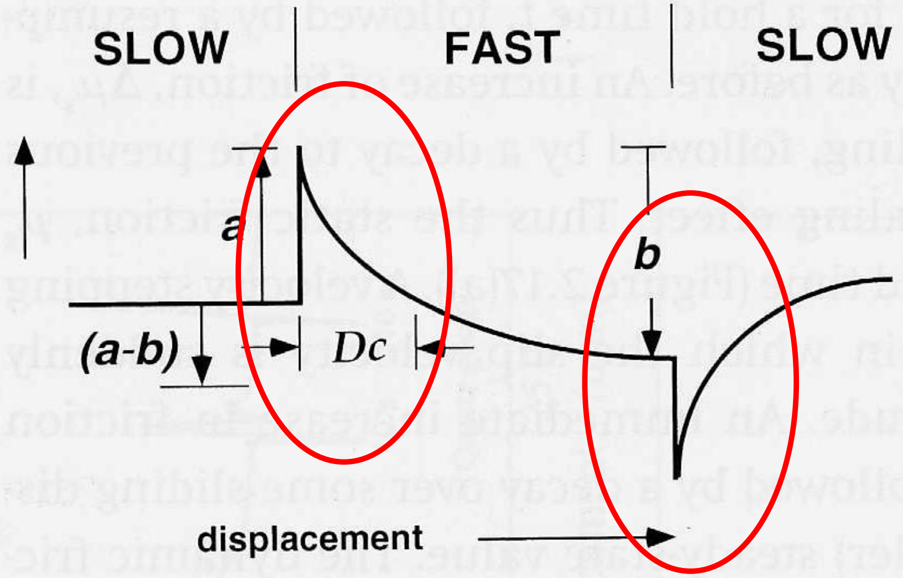


Fig. 2.18. Schematic diagram showing the response to *e*-fold velocity changes and defining the terms in the rate-state friction law.

(Scholz, 1998)

$$\bar{\sigma} = \sigma - p$$

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$$\boldsymbol{\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c}}$$

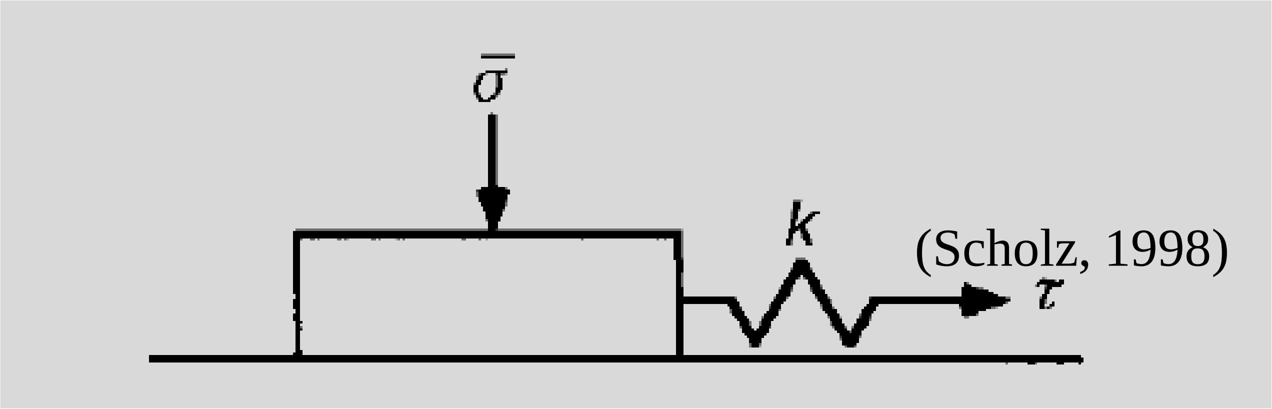
(Dieterich, 1981)

Effect of State Parameter

$(a-b) > 0$ velocity strengthening (stable sliding)

$(a-b) < 0$ velocity weakening (potentially unstable sliding)

Stability



(Scholz, 1998)

Stability

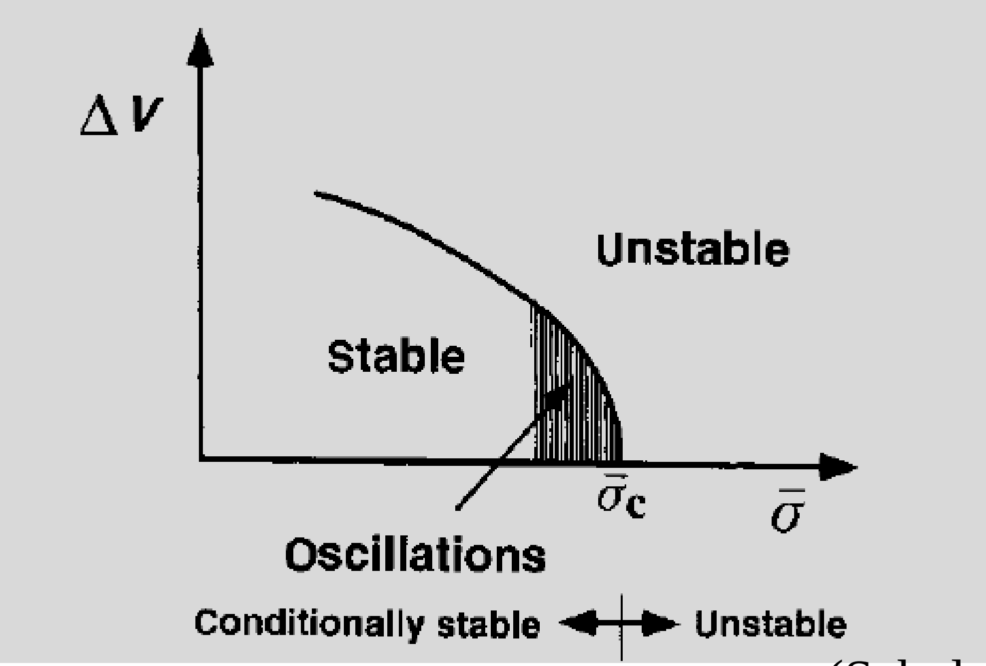
Frictional stability depends entirely on $\bar{\sigma}, \tau, k$, the friction parameters Dc and (a-b), and is independent of base friction μ_0.

Stability

Considering fixed stiffness k, oscillations occurs at a critical value of effective normal stress, σ_c, given by:

$$\sigma_c = \frac{kD_c}{-(a-b)}$$

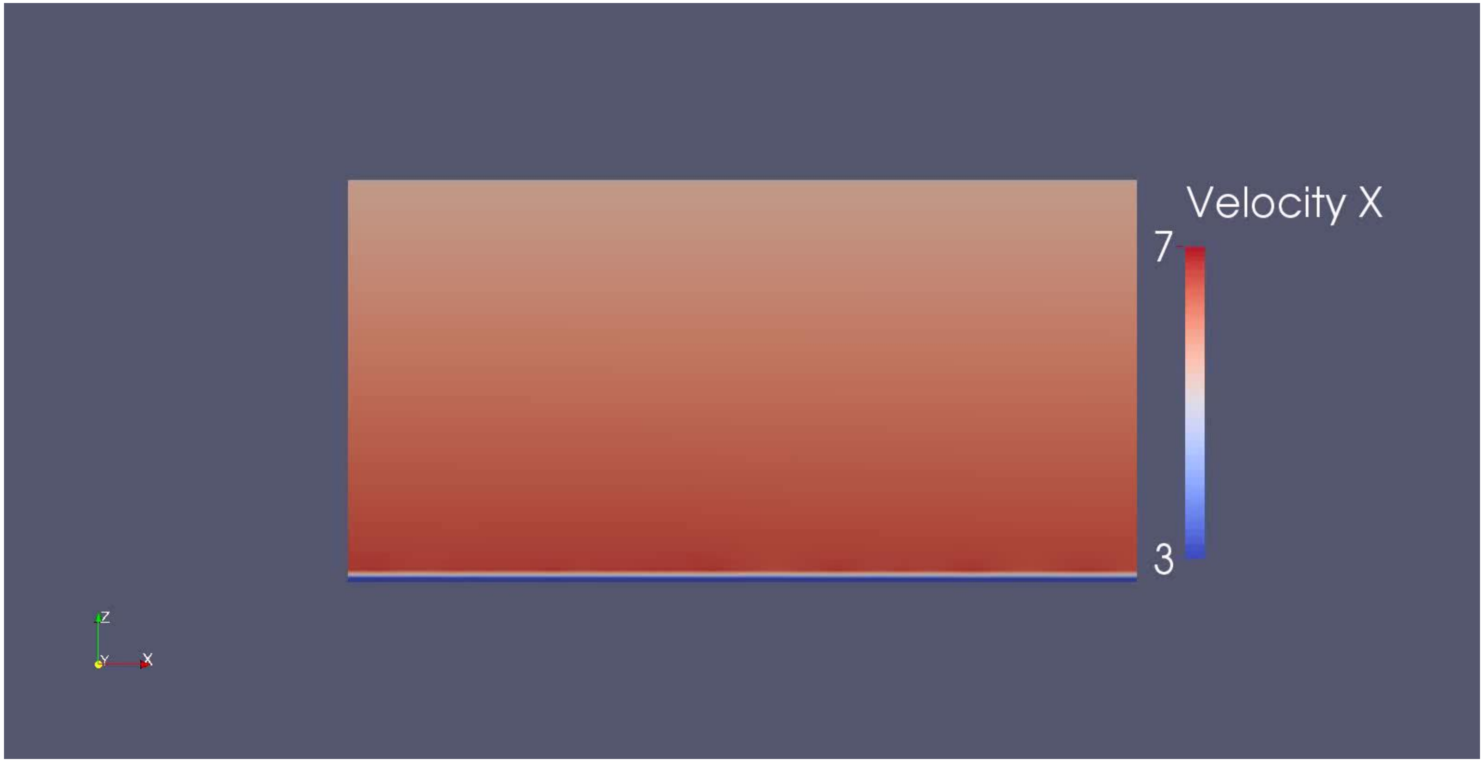
Stability



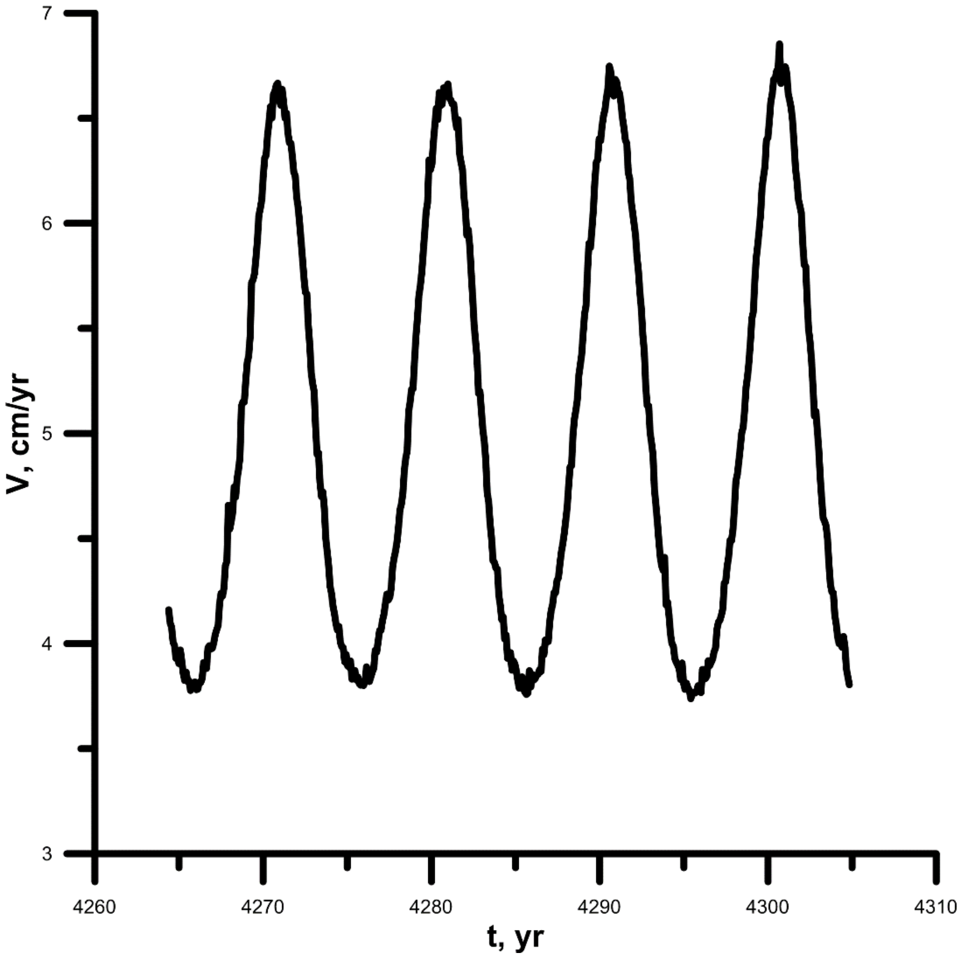
(Scholz, 1998)

$\sigma > \sigma_c$ *sliding is unstable*
$\sigma < \sigma_c$ *sliding is stable*
$\sigma \approx \sigma_c$ oscillations

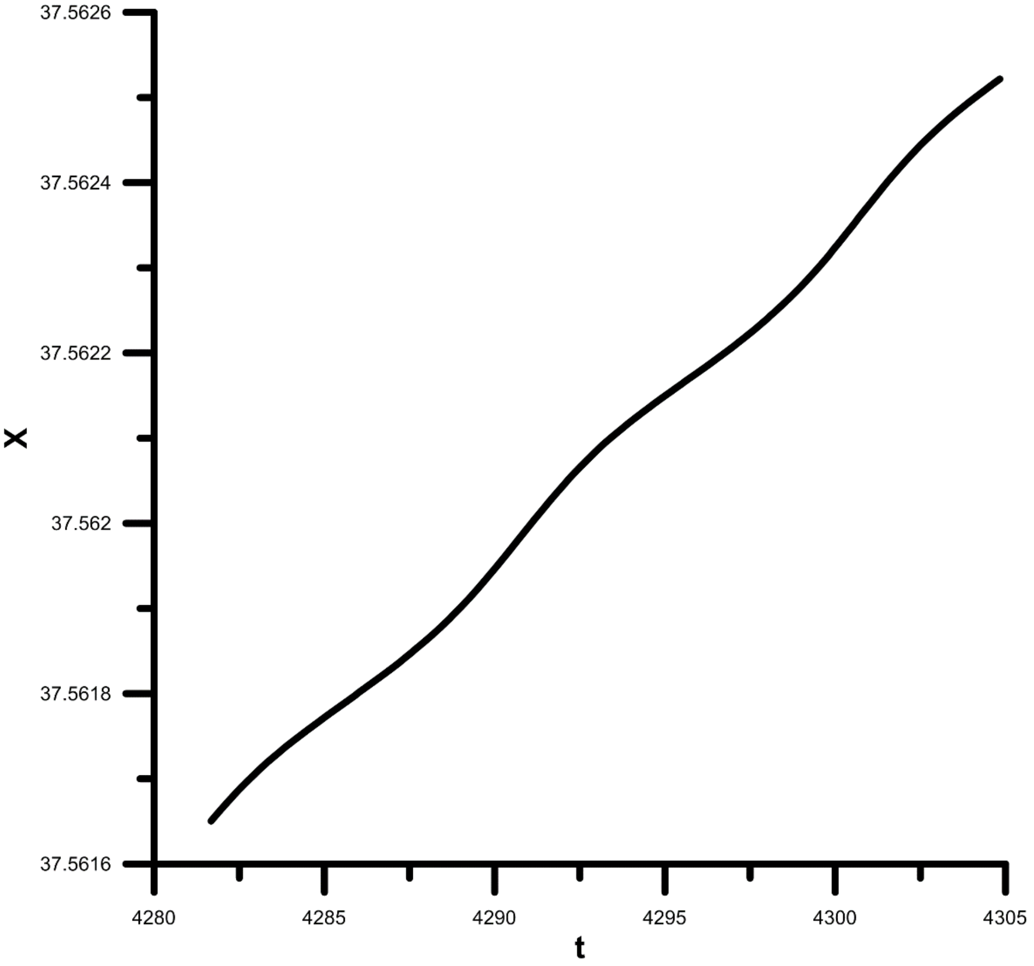
Oscillations Mode



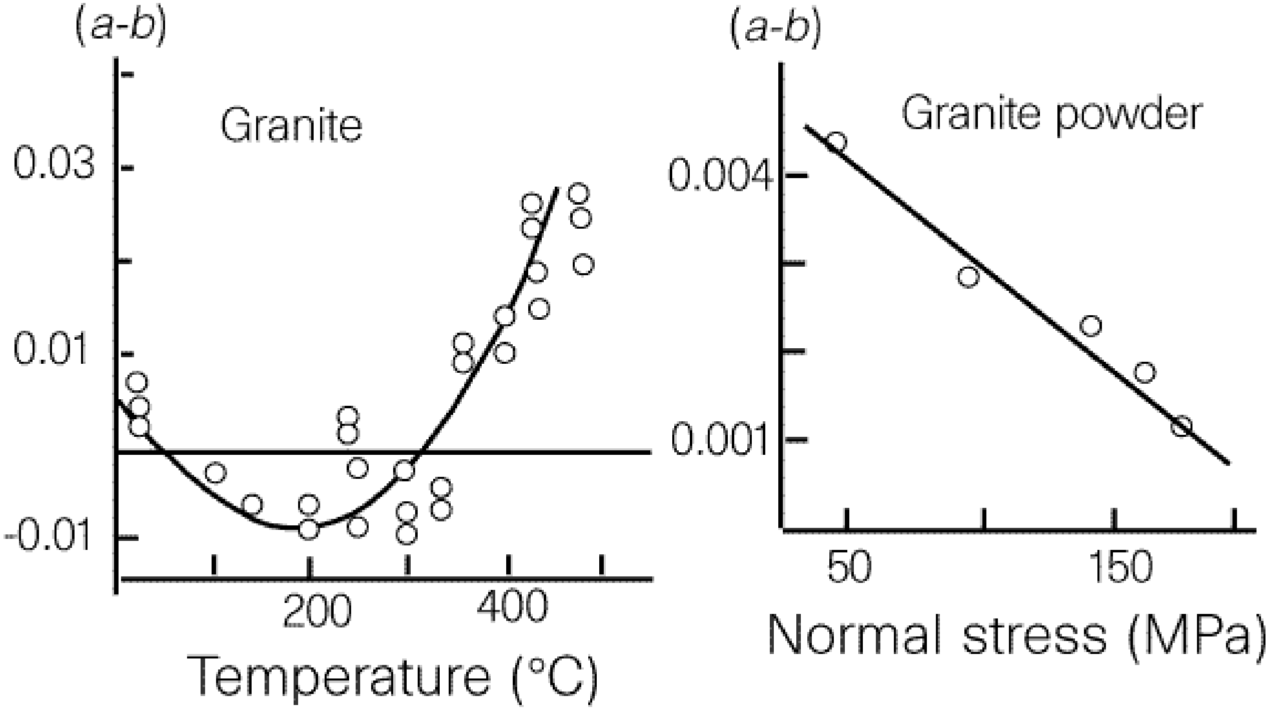
Oscillations Mode



Oscillations Mode

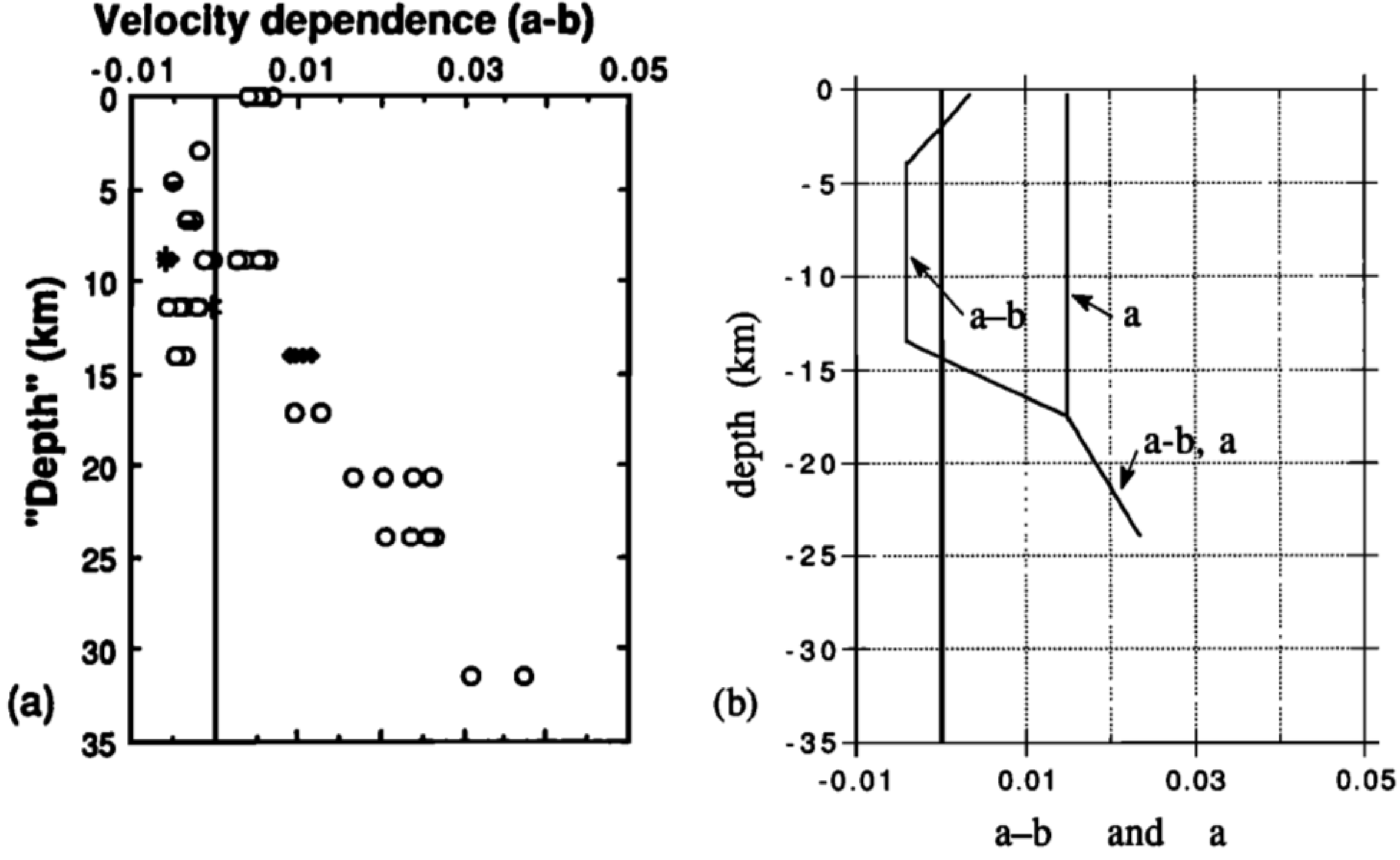


Variation of (a-b)



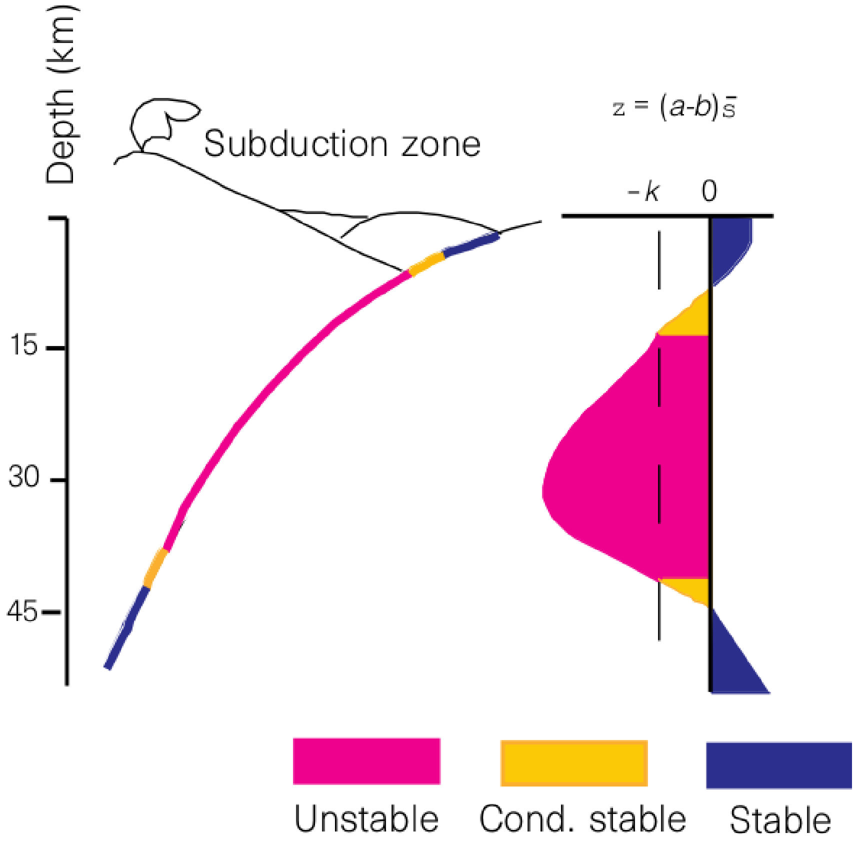
(Scholz, 1998)

Variation of (a-b)



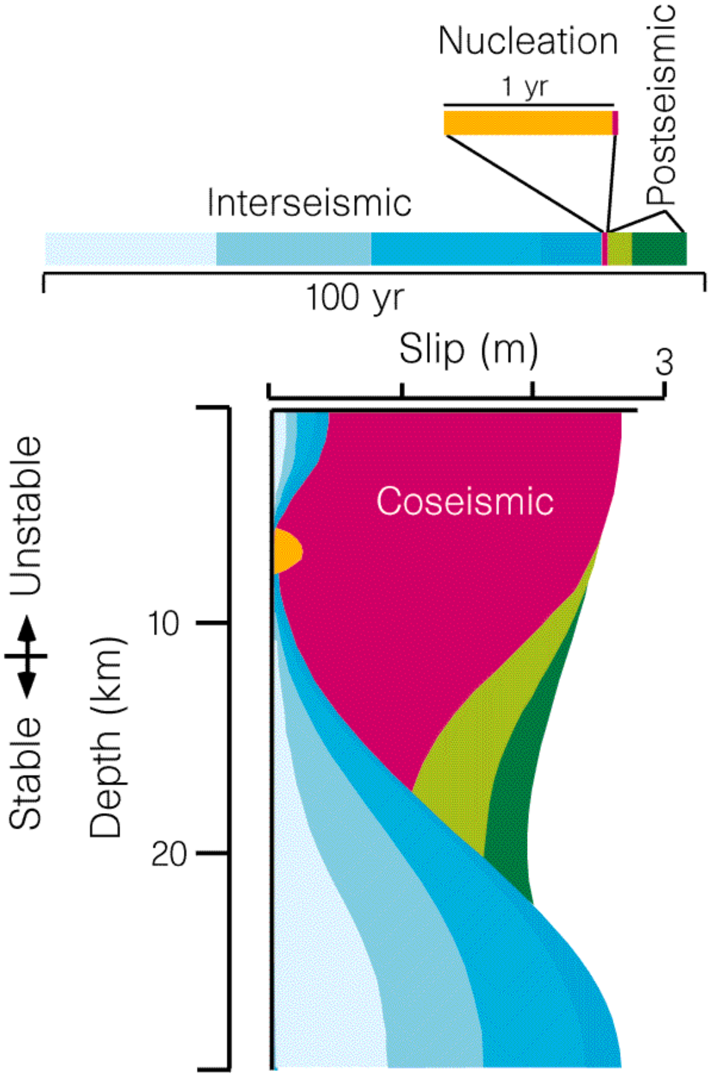
(Rice, 1993)

Variation of (a-b)



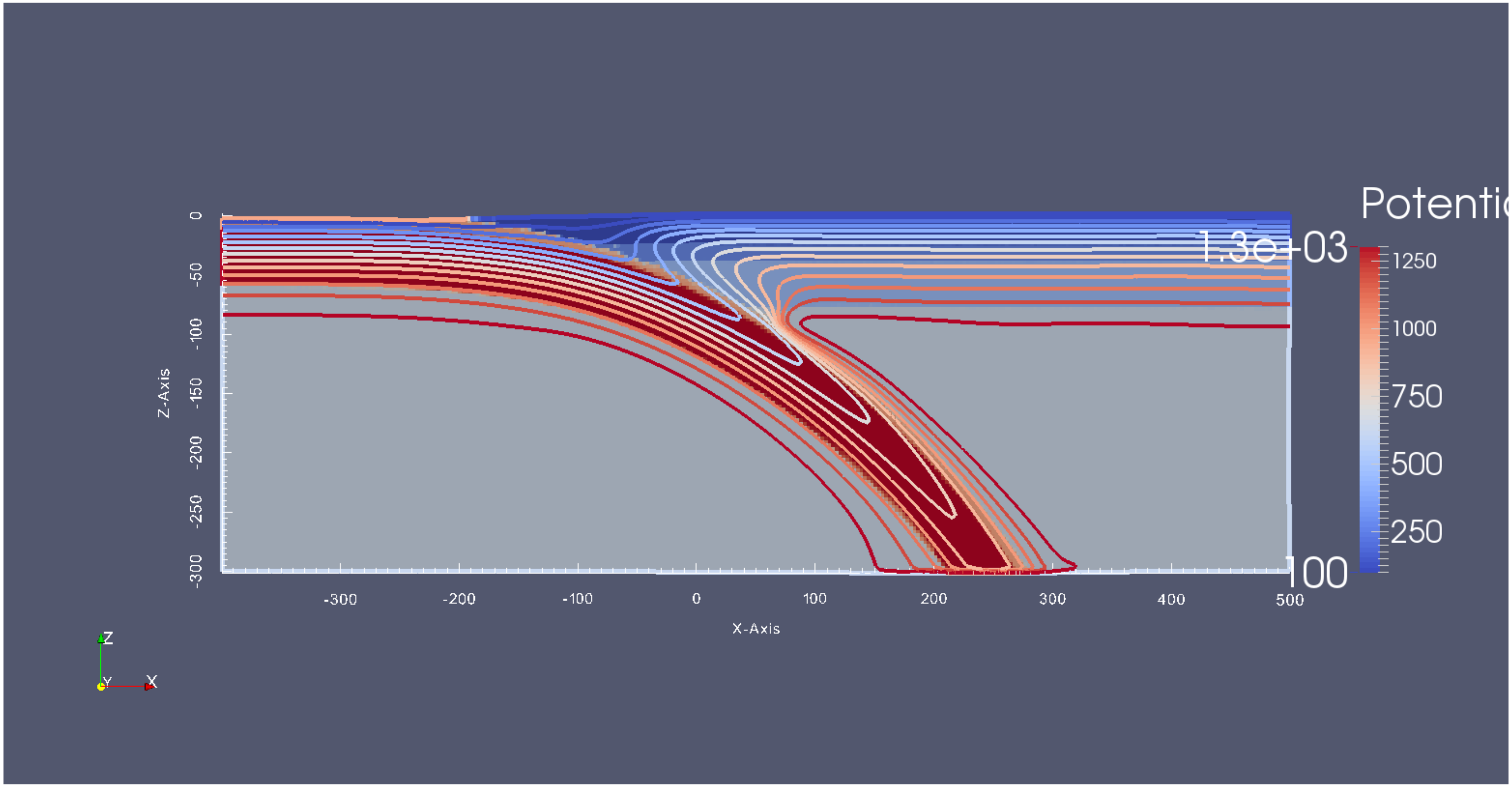
(Scholz, 1998)

Slip Distribution

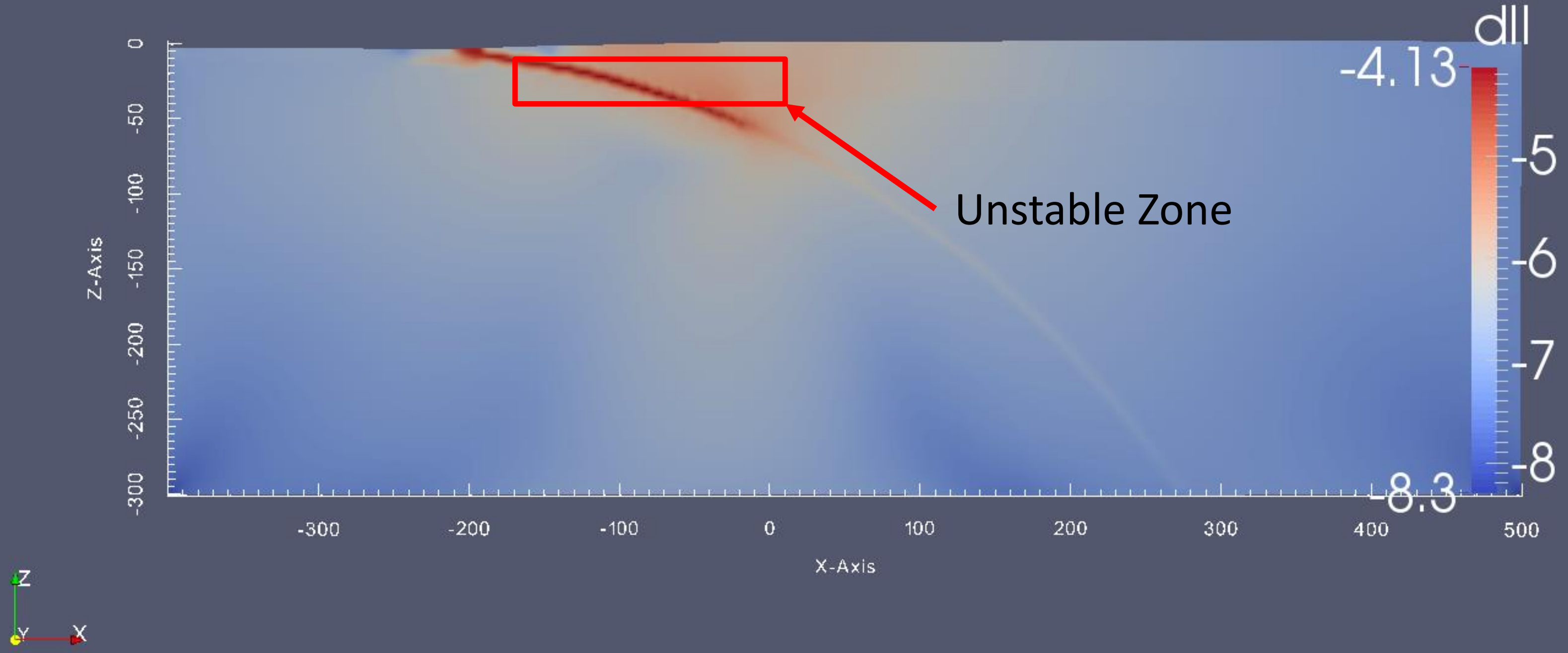


(Scholz, 1998)

Rupture in Subduction Zone



Rupture in Subduction Zone



Unstable Zone

Nucleation

$$\sigma_c = \frac{kD_c}{-(a-b)}$$

The effective stiffness, k, of a crack with length L embedded in an elastic medium with shear modulus G scales as G/L ($k = \eta G/L$).

$$L_c = \frac{G\eta D_c}{(b-a)\bar{\sigma}}$$

Nucleation

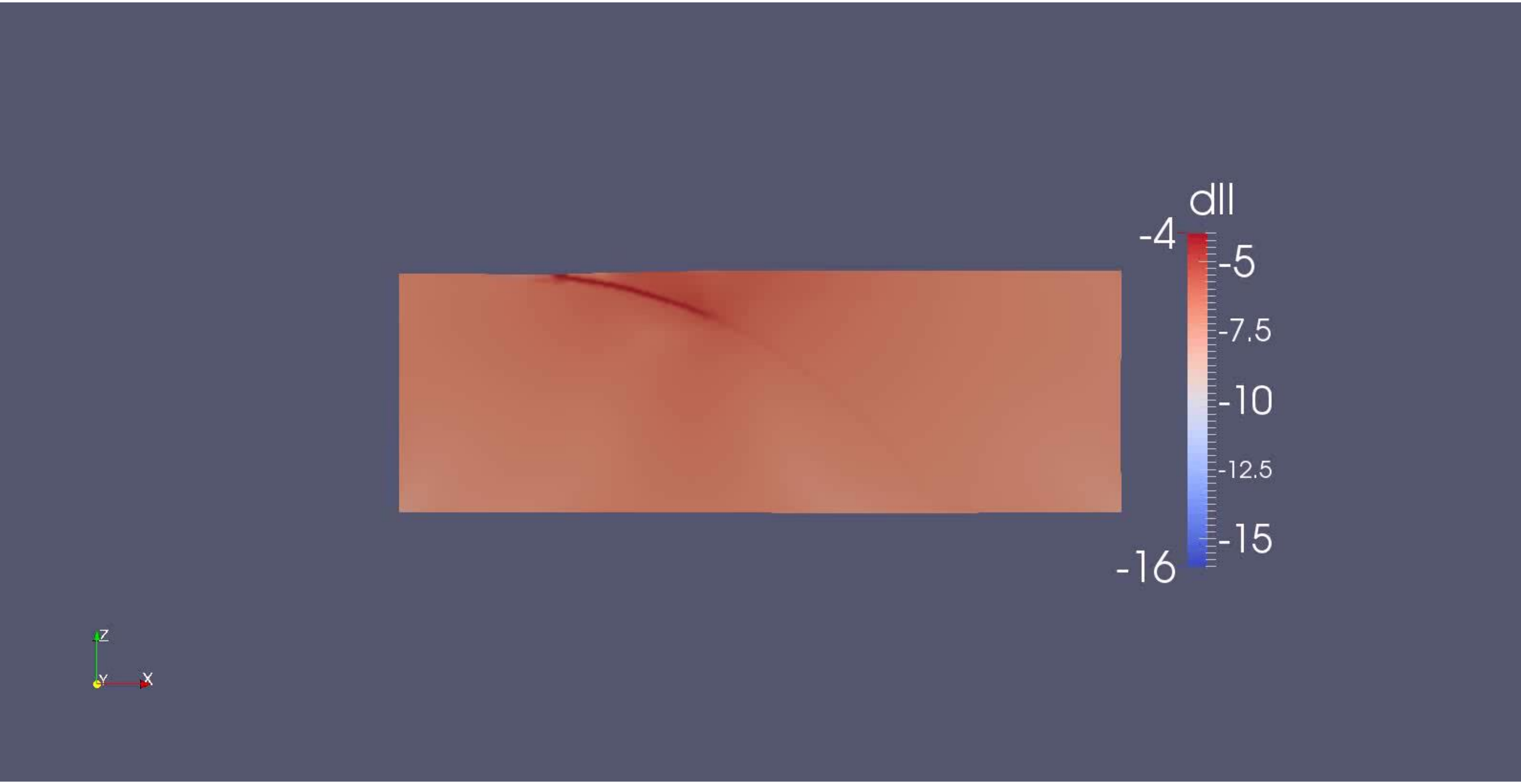
$$L_c = \frac{G\eta D_c}{(b-a)\bar{\sigma}}$$

(Scholz, 1998)

- Instability occurs when the slipping patch reaches a critical size L_c.

- Also it is a minimal size of asperity to generate an earthquake.

Nucleation

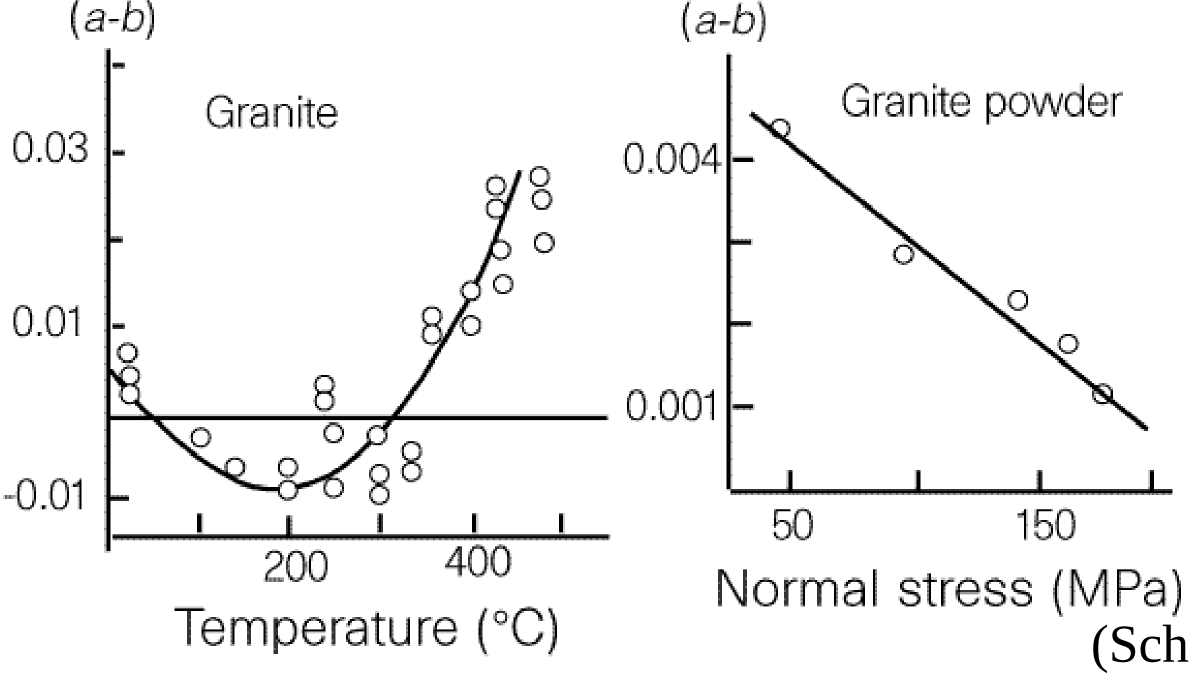


Values and Uncertainties

$a, b \sim 10^{-3} - 10^{-2}$

$(a - b) \sim 0.004$ (in seismogenic zone)

- Varies with stress and temperature.
- Hard to model in experiment.



(Scholz, 1998)

Values and Uncertainties

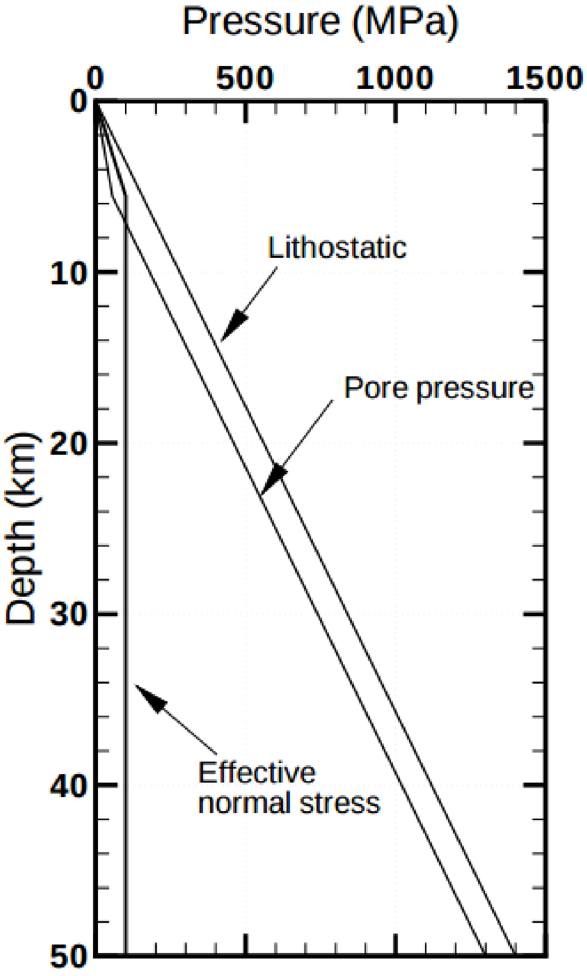
$D_c \approx 1 - 100\ \mu m$ in the laboratory experiments

But various attempts to model D_c, assuming that it is a property of the surface contact topography or gouge zone thickness, suggest that it may be much larger at the fault scale (up to 1m)

Values and Uncertainties

Effective stress distribution

$$\bar{\sigma} \sim 50 MPa$$

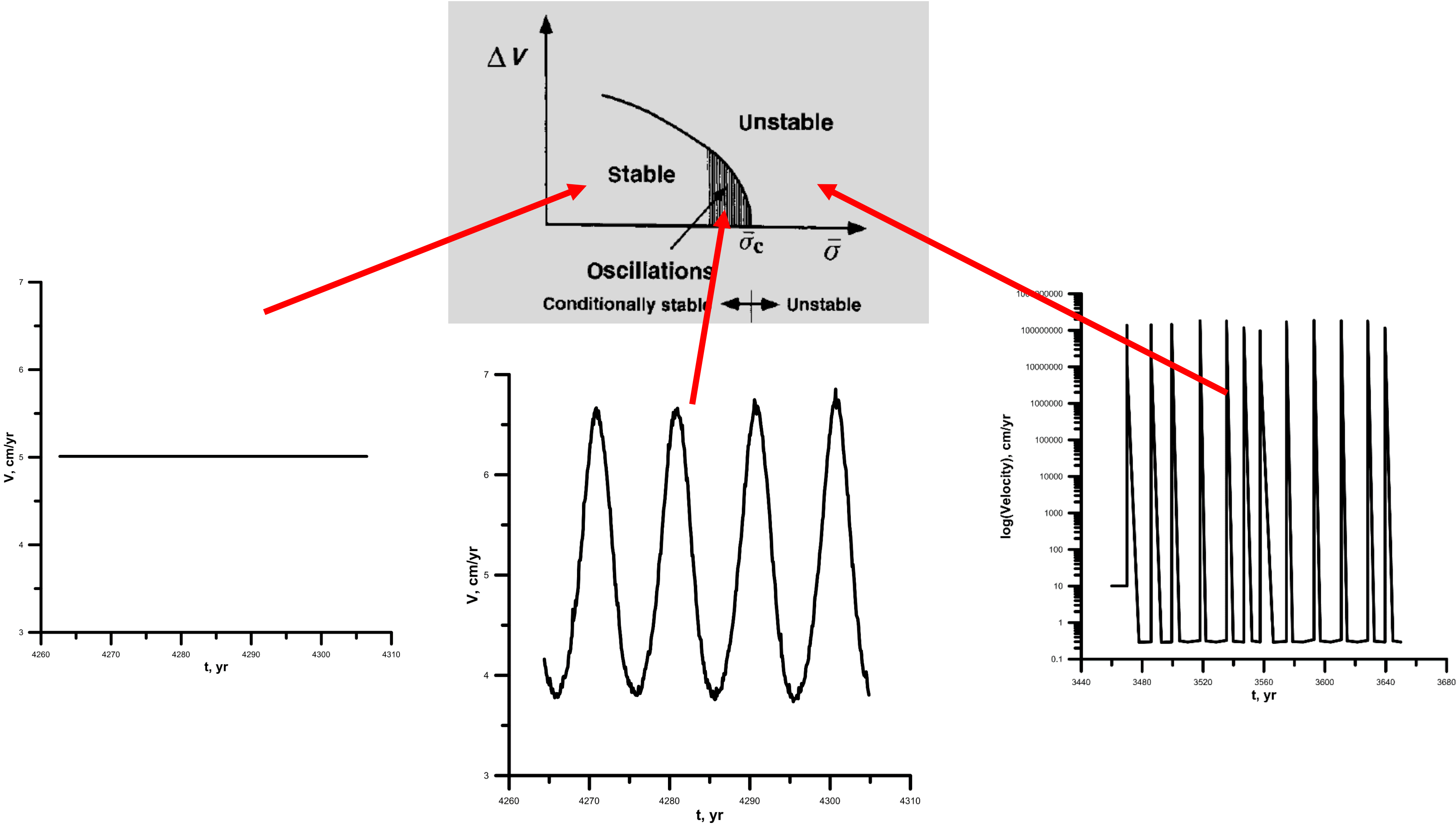


Conclusons

$(a-b) > 0$ velocity strengthening (stable sliding)

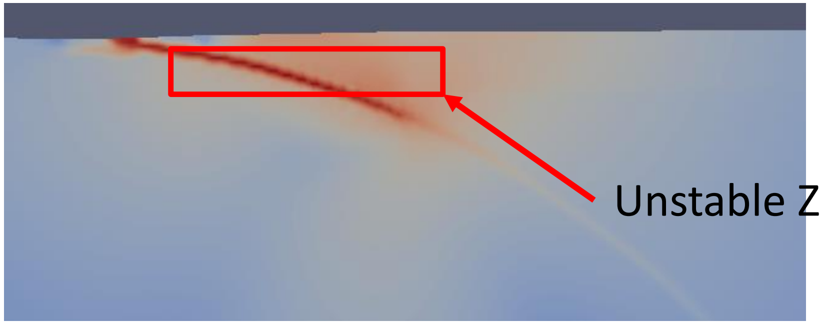
$(a-b) < 0$ potentially velocity weakening (unstable sliding)

Conclusions

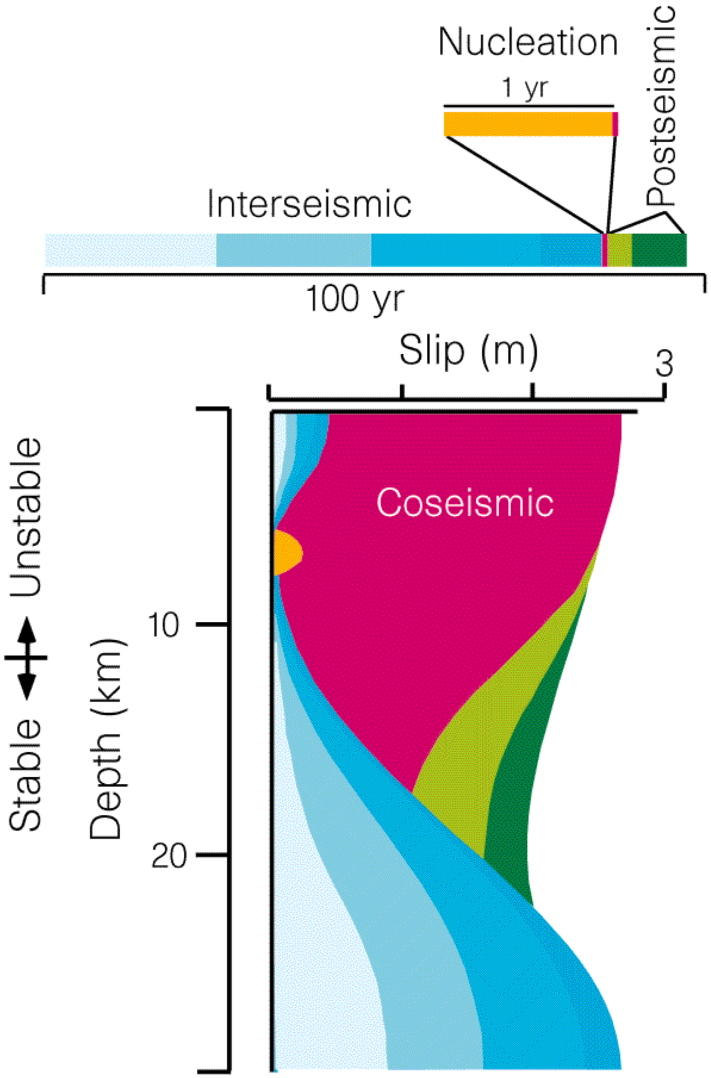


Conclusions

Earthquakes occur in the unstable field but can also propagate to conditionally stable fields



Unstable Zone



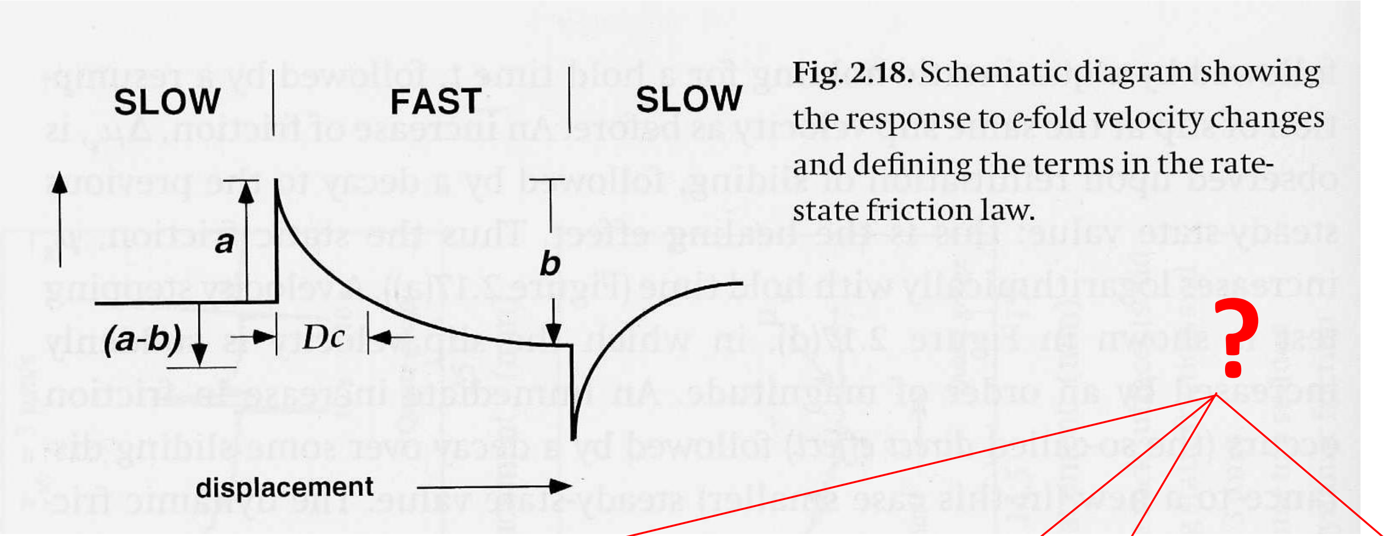
Conclusions

Nucleation size is necessary for earthquake

$$L_c = \frac{G\eta D_c}{(b-a)\bar{\sigma}}$$

Conclusions

Uncertainties for friction parameters



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(Dieterich, 1981)