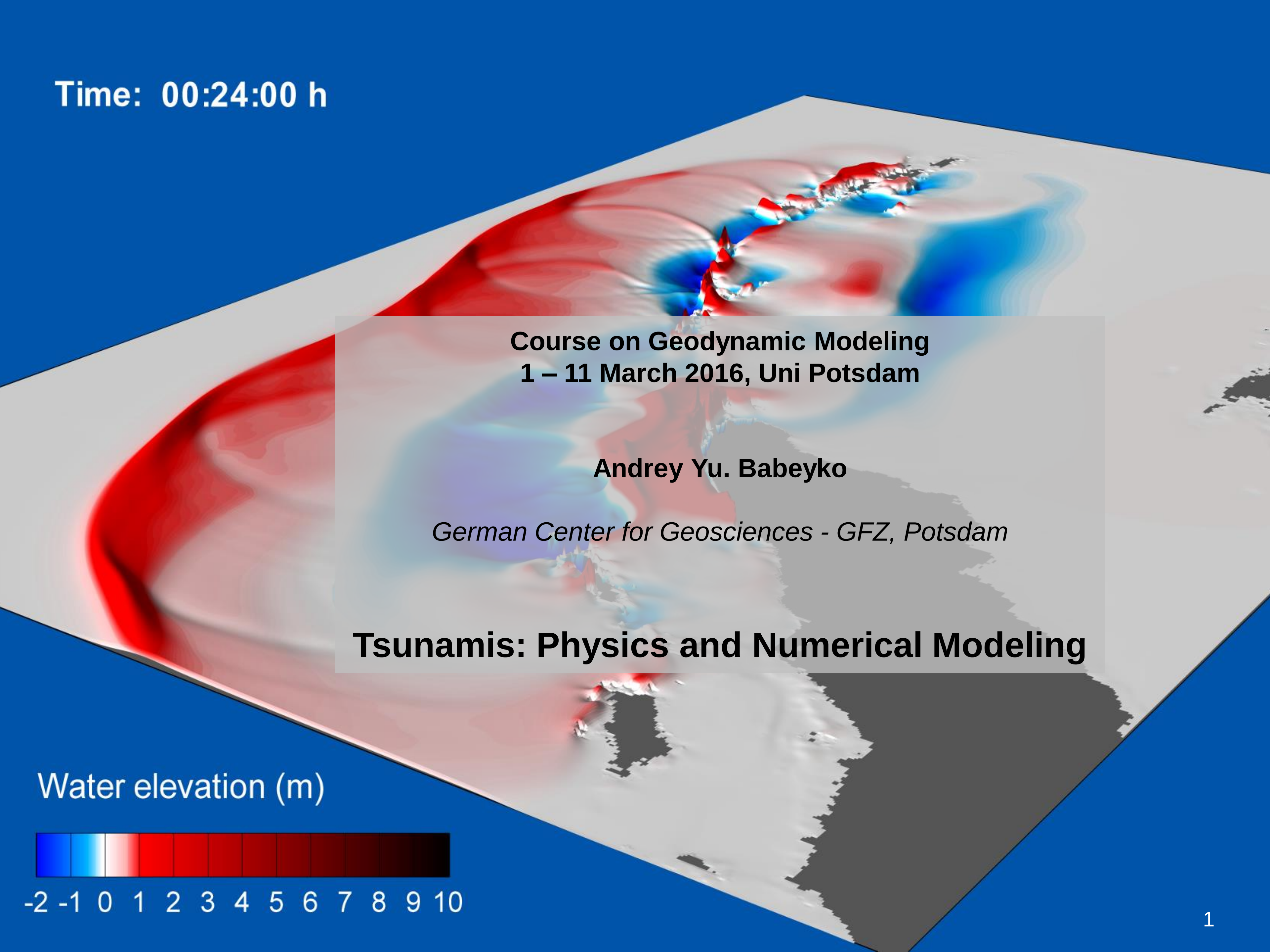
**Course on Geodynamic Modeling**
**1 – 11 March 2016, Uni Potsdam**

**Andrey Yu. Babeyko**

*German Center for Geosciences - GFZ, Potsdam*

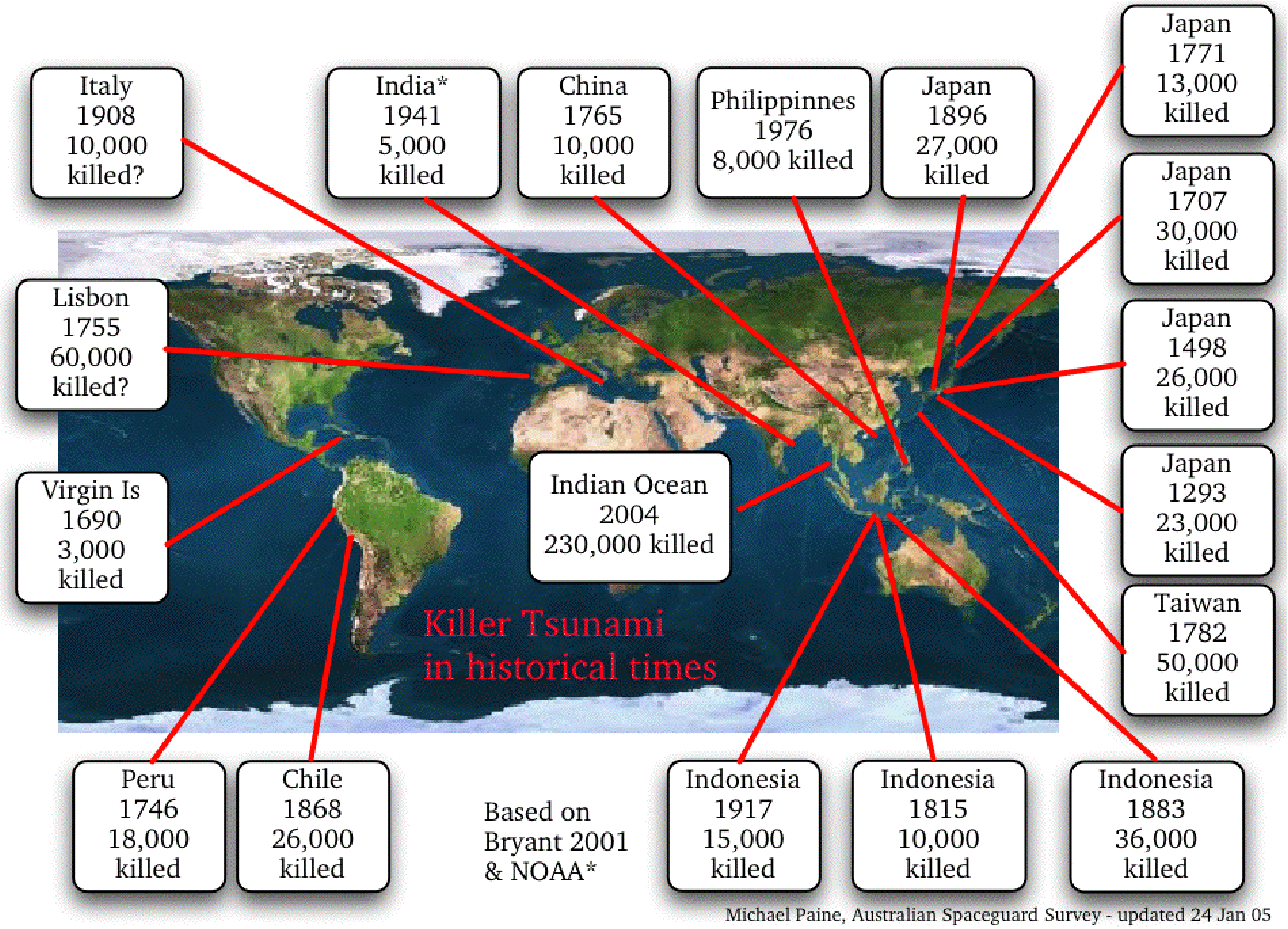
# Tsunamis: Physics and Numerical Modeling



**Outline**:

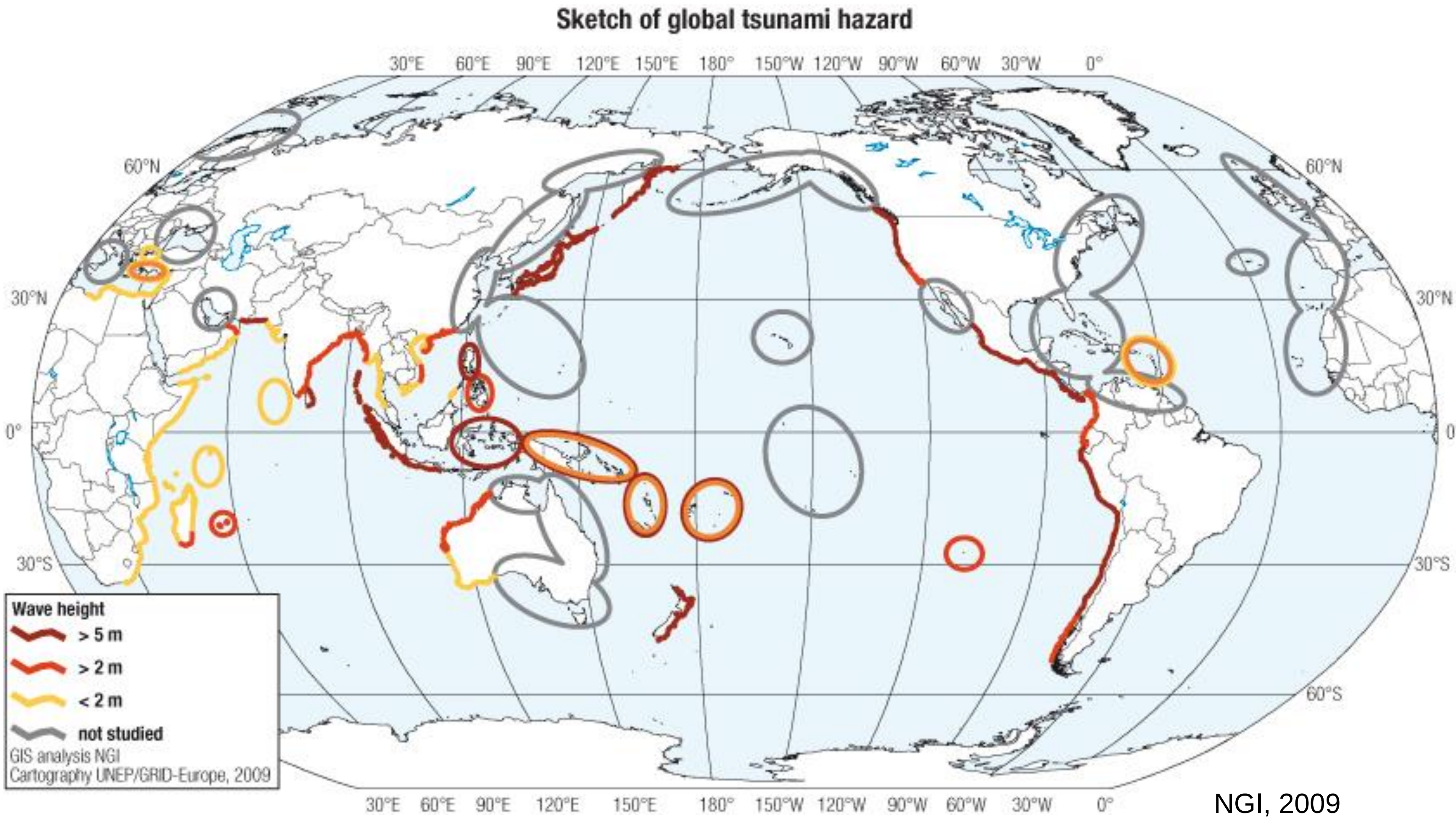
1. Tsunami as global natural hazard
2. Tsunamigenic sources
3. Tsunami modeling: background
4. Tsunami modeling: applications

# Tsunami as global hazard



Michael Paine, Australian Spaceguard Survey - updated 24 Jan 05

# Global Assessment Report on Disaster Risk Reduction -- ***GAR***



NGI, 2009

# Global Assessment Report on Disaster Risk Reduction -- ***GAR***

**Figure 2.30:** Number of people living in areas potentially affected by tsunamis

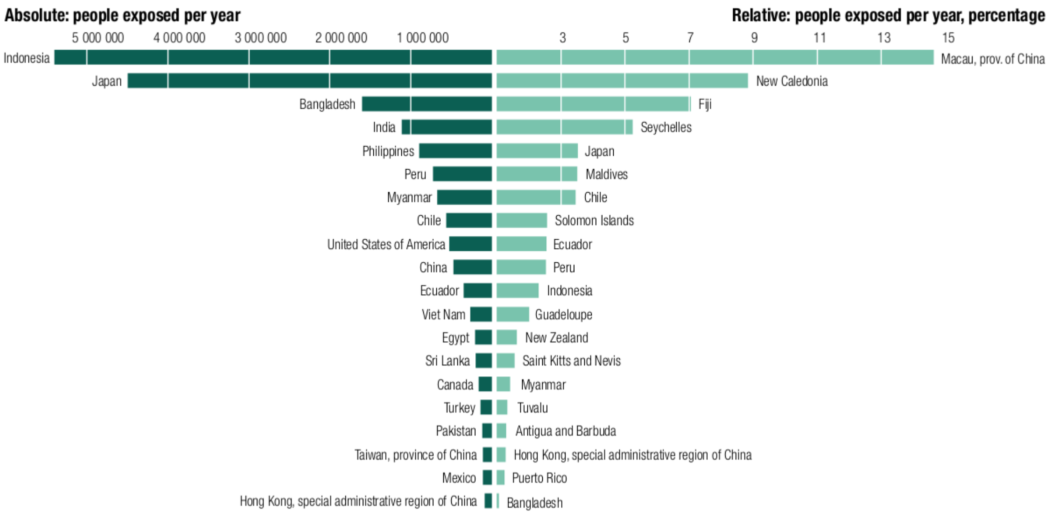
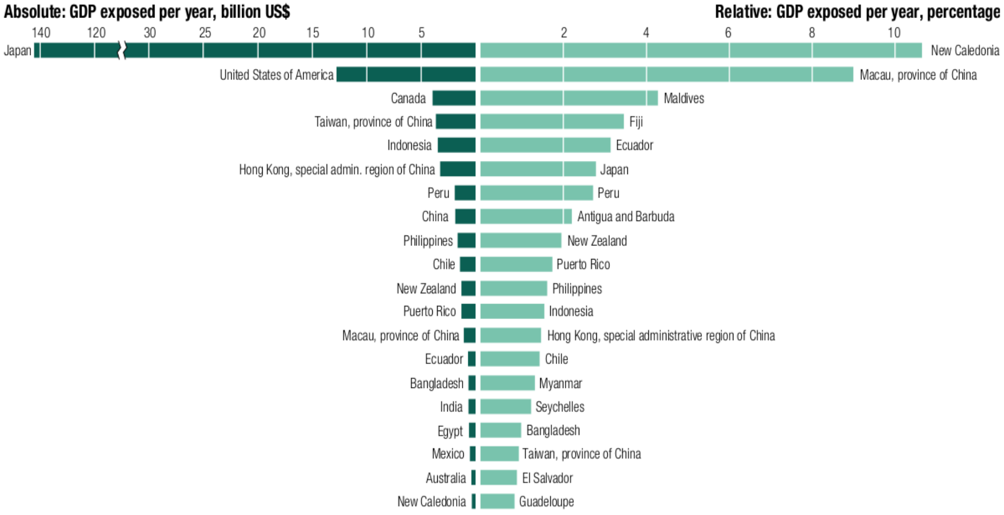


**Figure 2.31:** GDP exposed to tsunamis



NGI, 2009

# What controls Tsunami distribution worldwide?

## Lithospheric Plates and Distribution of Earthquakes

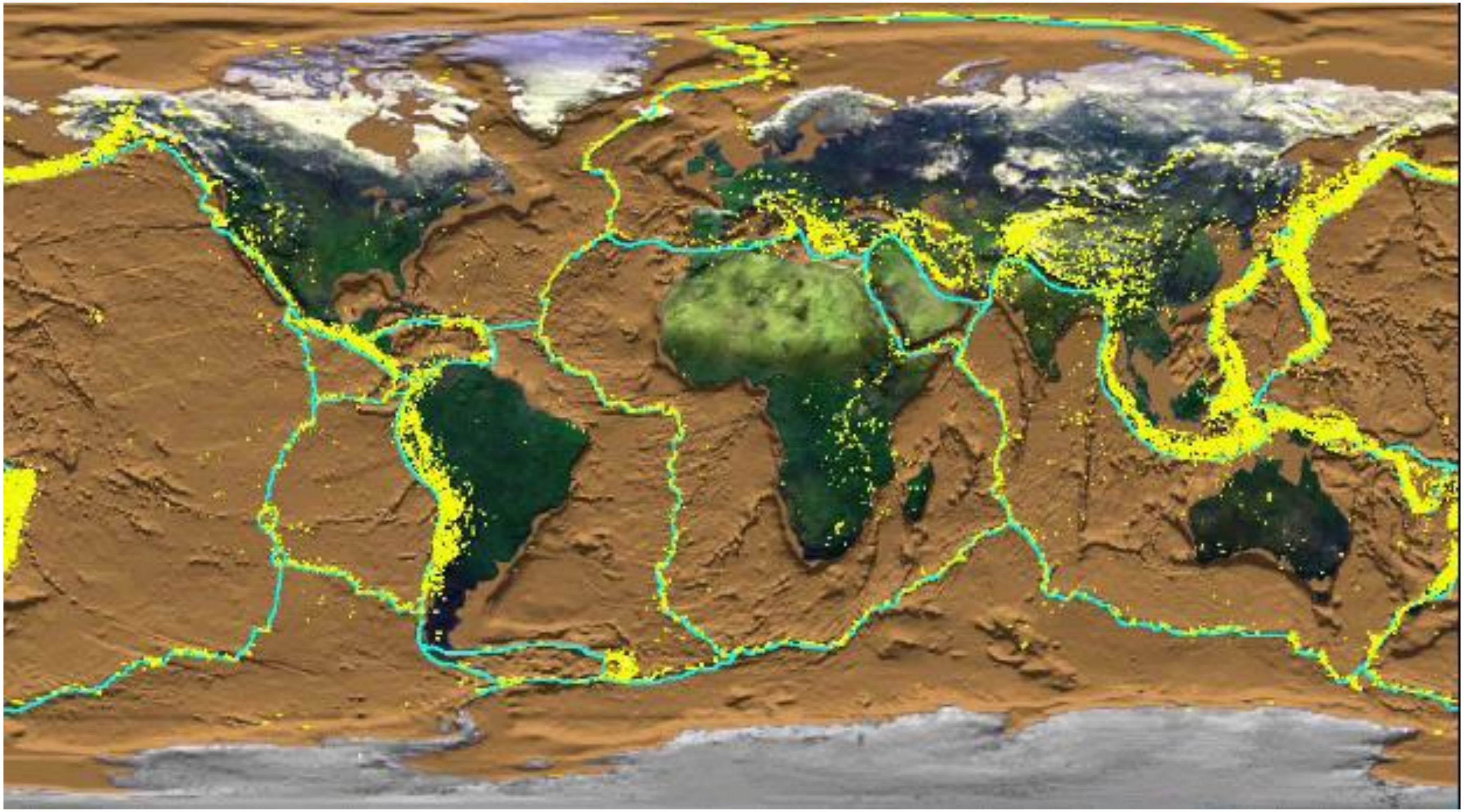
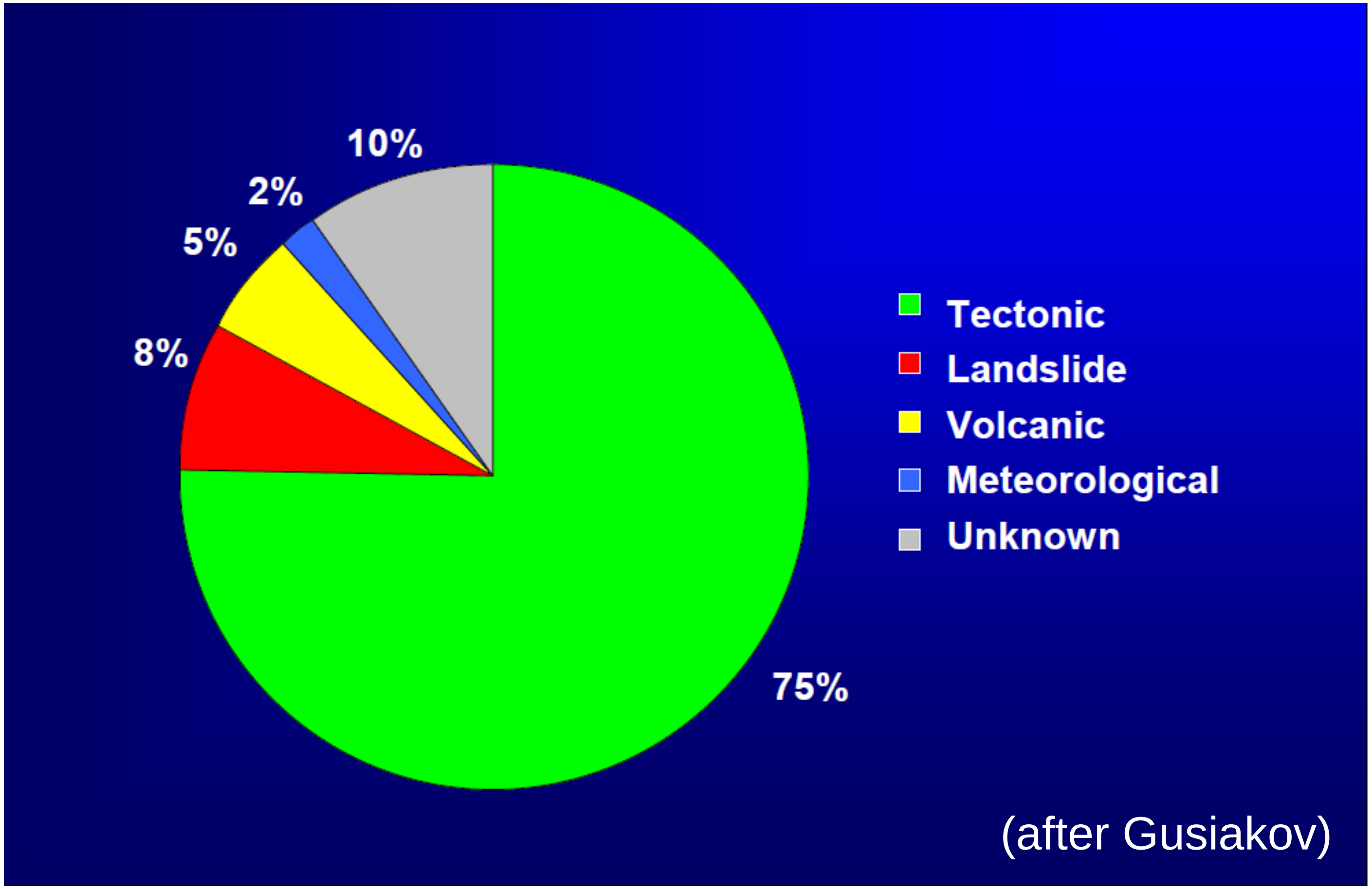


Image from Humboldt University Berlin

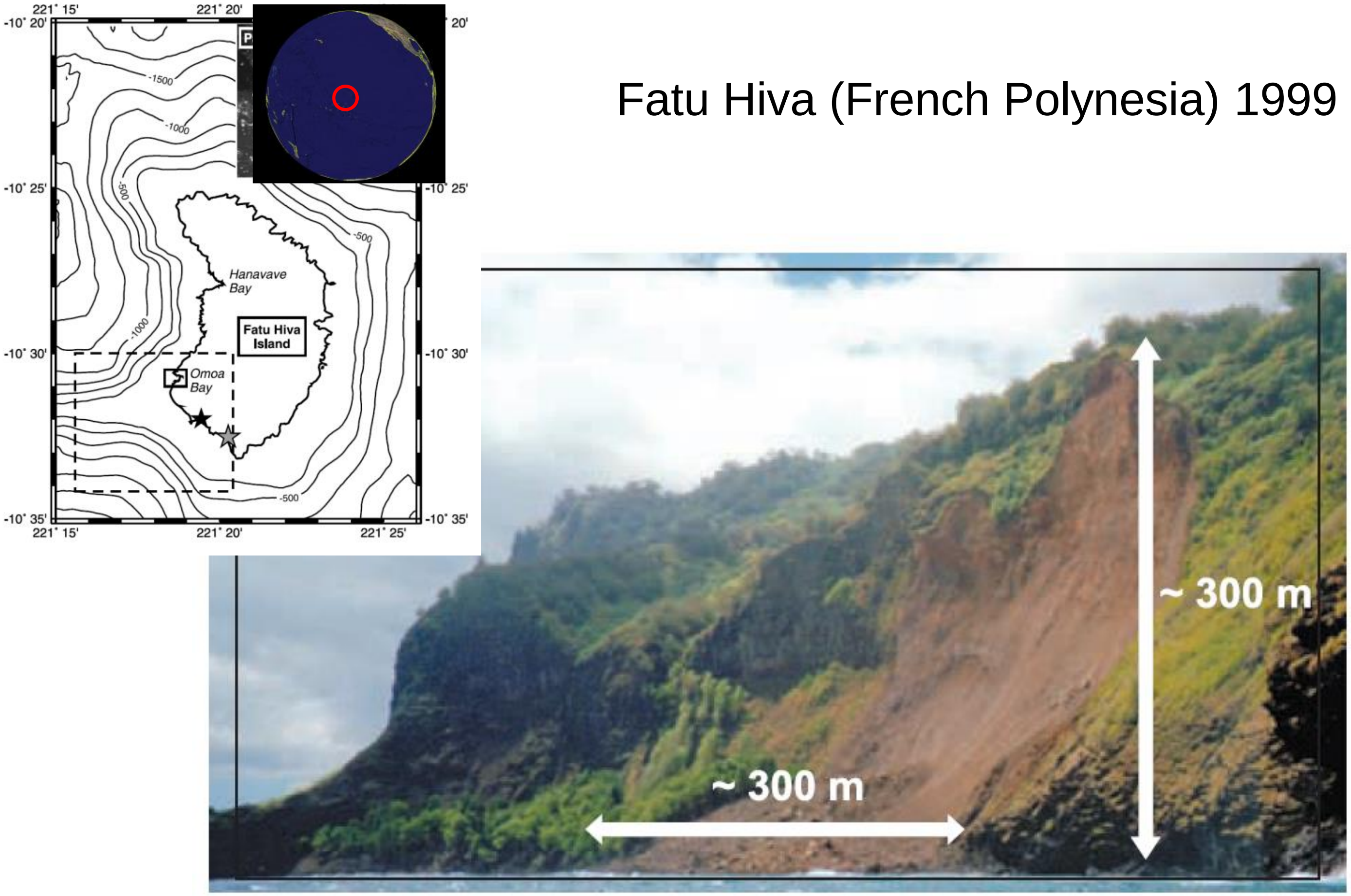
# Tsunamigenic Sources



(after Gusiakov)

# Landslides

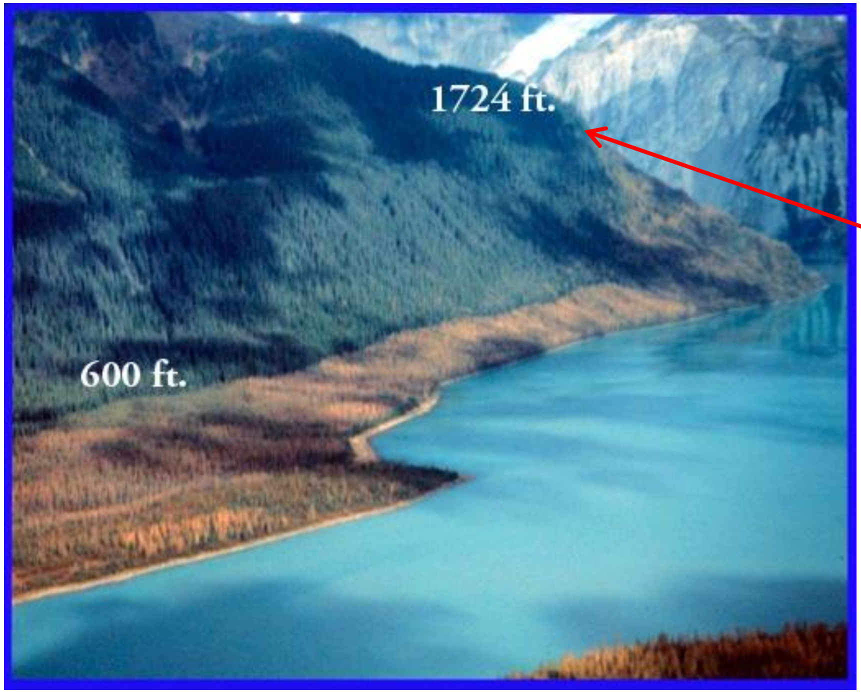
## Fatu Hiva (French Polynesia) 1999



**Hebert et al. (2002)**

# Landslides

Lituya Bay (Alaska, USA) 1958



About 500 m run-up

after G. Pararas-Carayannis

# Storegga Slide (Norway) 8200 B.C.

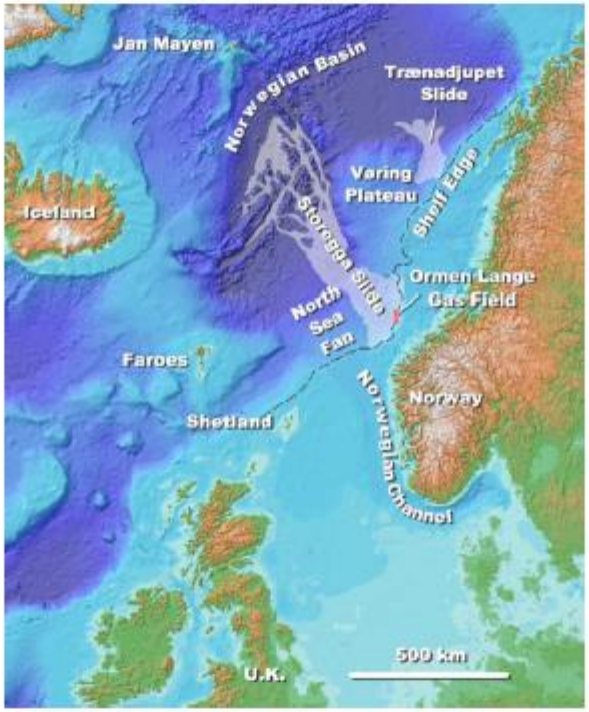
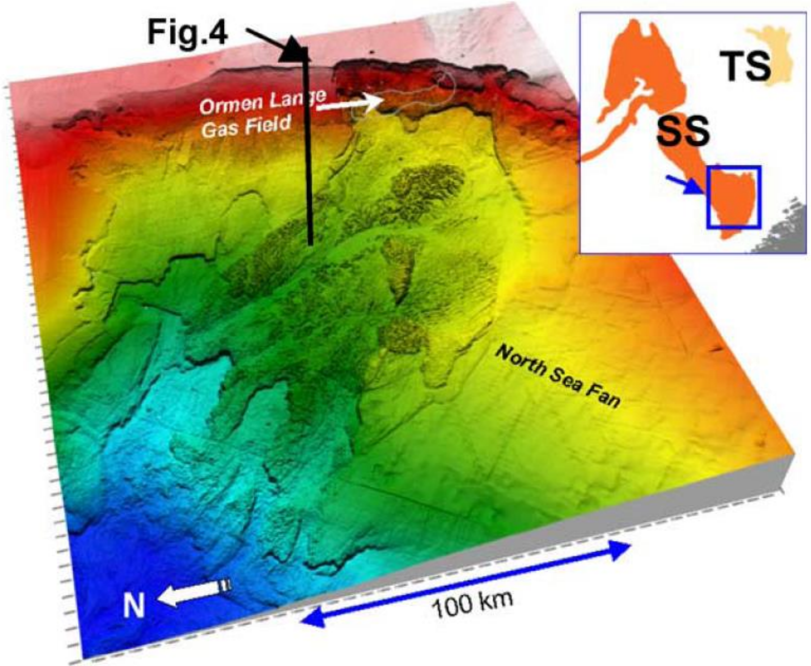


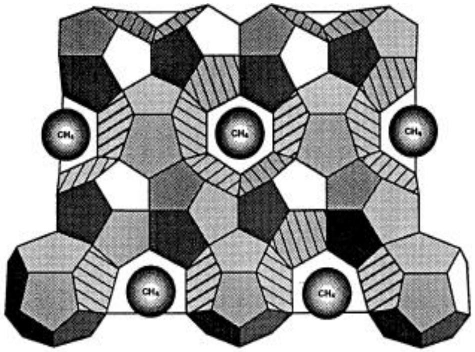
Fig. 1. Location map showing the Storegga and Trænadjupet submarine slides on the Mid-Norway margin.



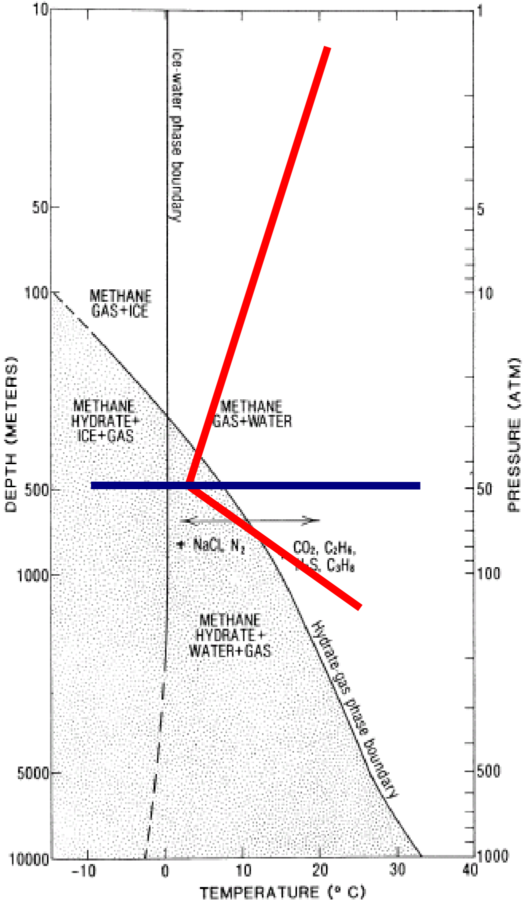
**Bryn et al. (2005)**

One of the biggest historic slides (2400 km$^3$).
Slides of this size are extremely rare but re-occur in geological time scales.

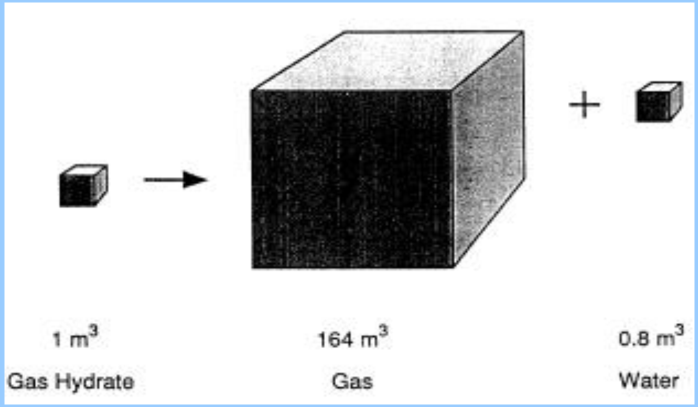
# Gas hydrates can destabilize submarine slopes



Natural gas hydrates are solid crystalline compounds composed of molecules of natural gas trapped in cages of water molecules.
Looks like ice and has similar density.

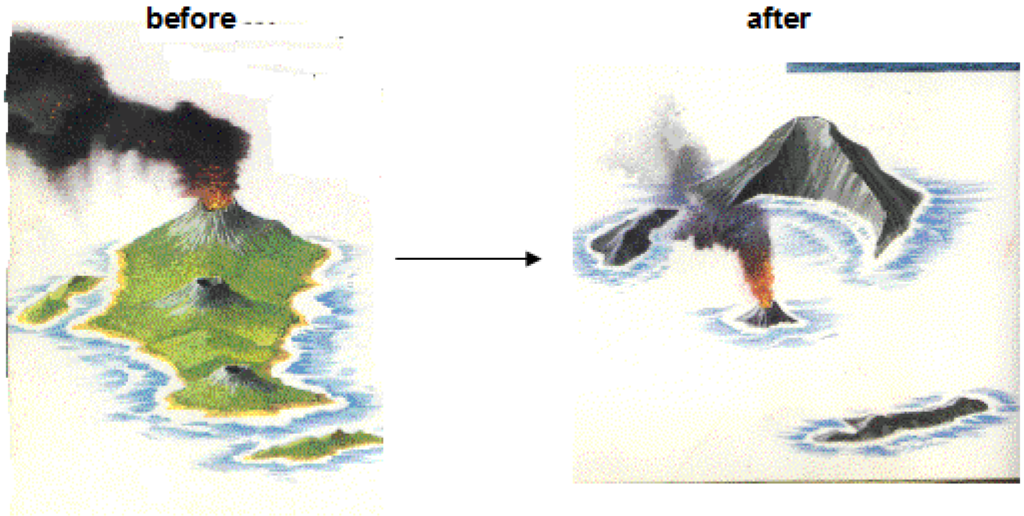


That's why they are important for us!

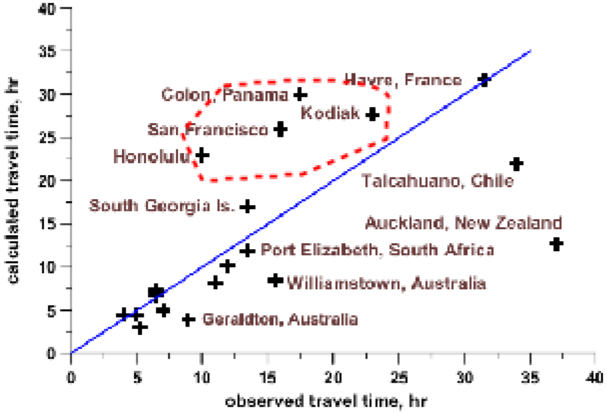


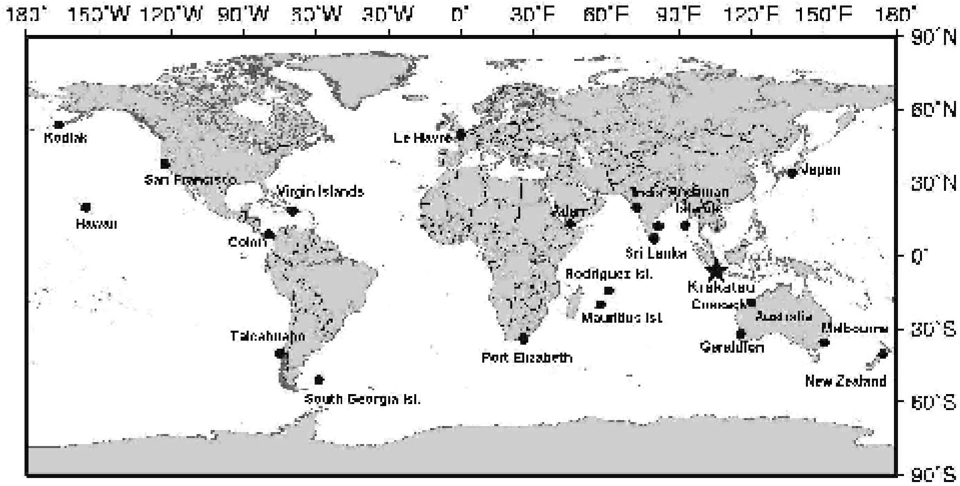
# Volcanic eruptions

## Krakatau, August 26, 1883



after G. Pararas-Carayannis
(www.drgeorgepc.com)





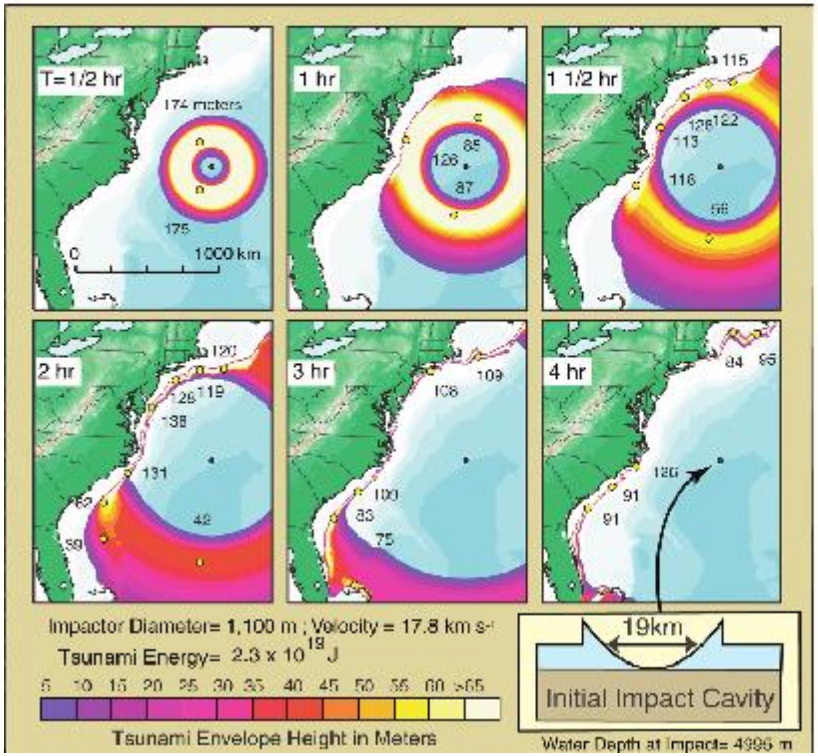
Sea-level change recordings and analysis (Choi et al., 2003)

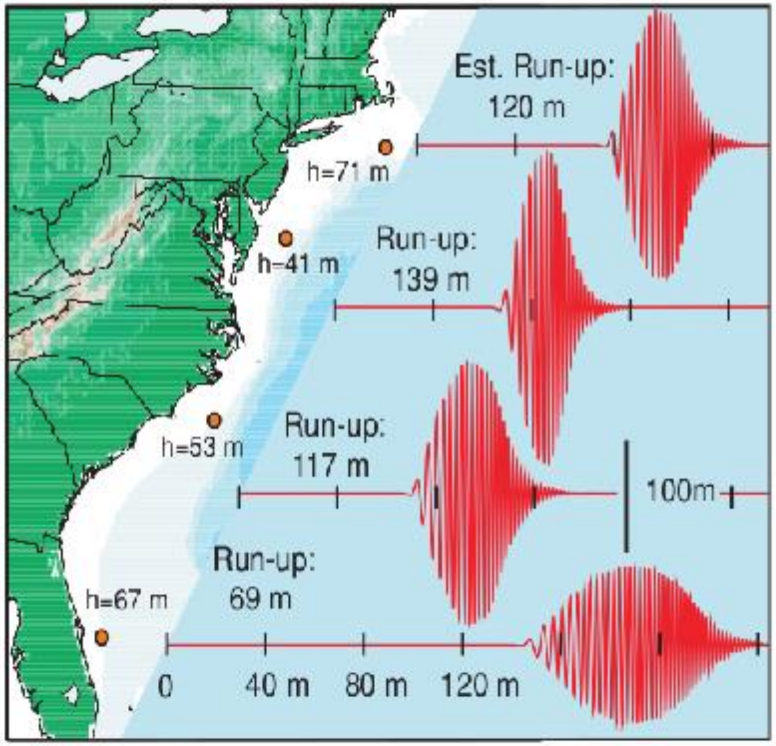
# Asteroid impacts

## Asteroid impact tsunami of 2880 March 16

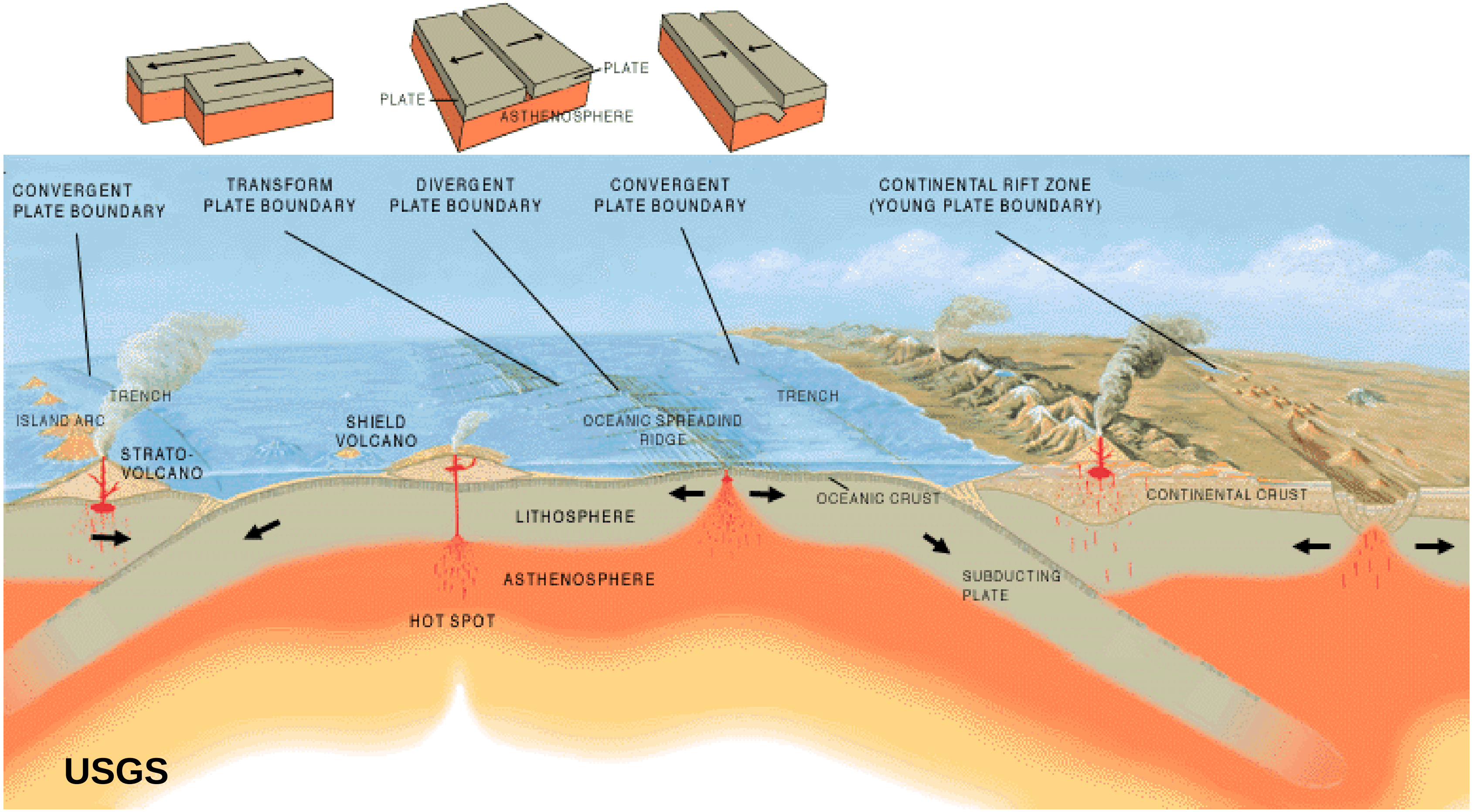
A scenario according to Ward & Asphaug (2003)

A 1.1-km diameter asteroid named „*DA 1950*" has a 0.0-0.3% probability to collide with the Earth in the year 2880



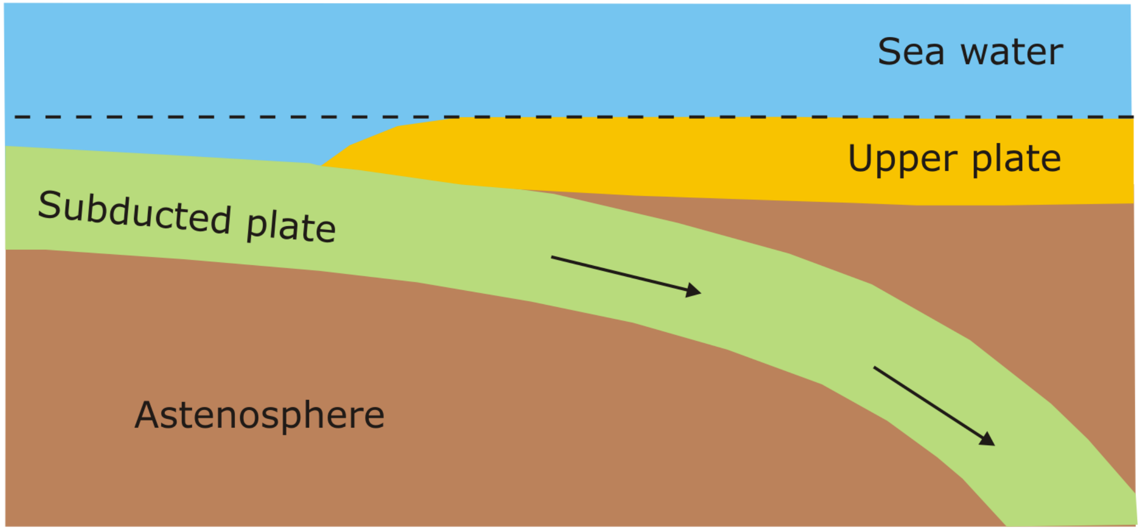


# Elements of plate tectonics

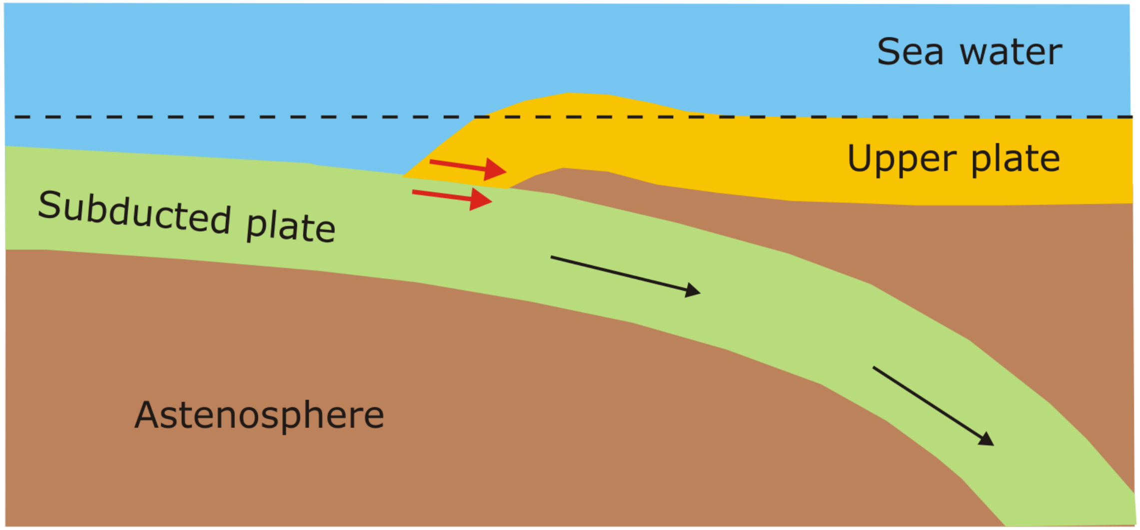


USGS

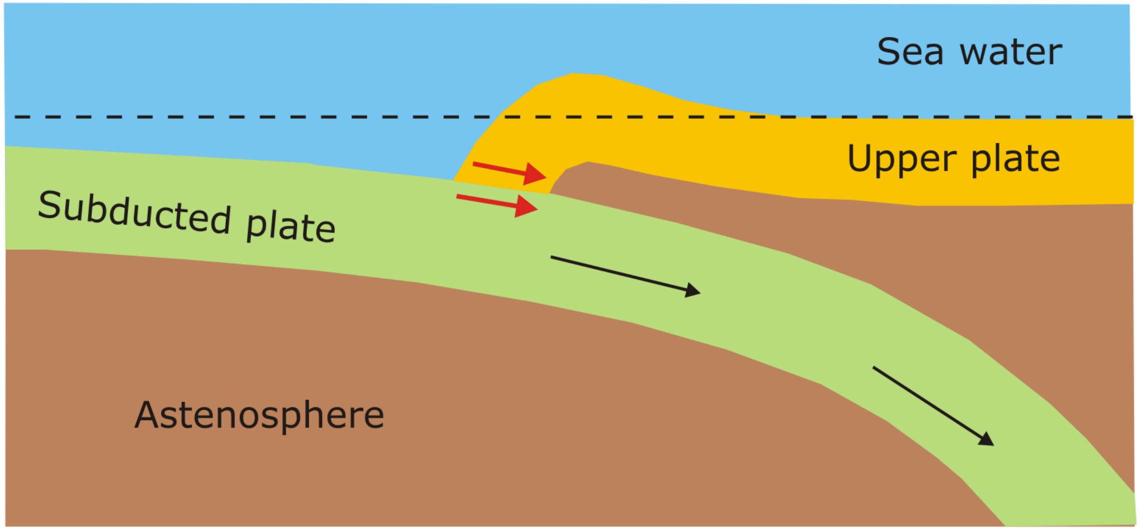
# Mechanism of interplate subduction earthquake



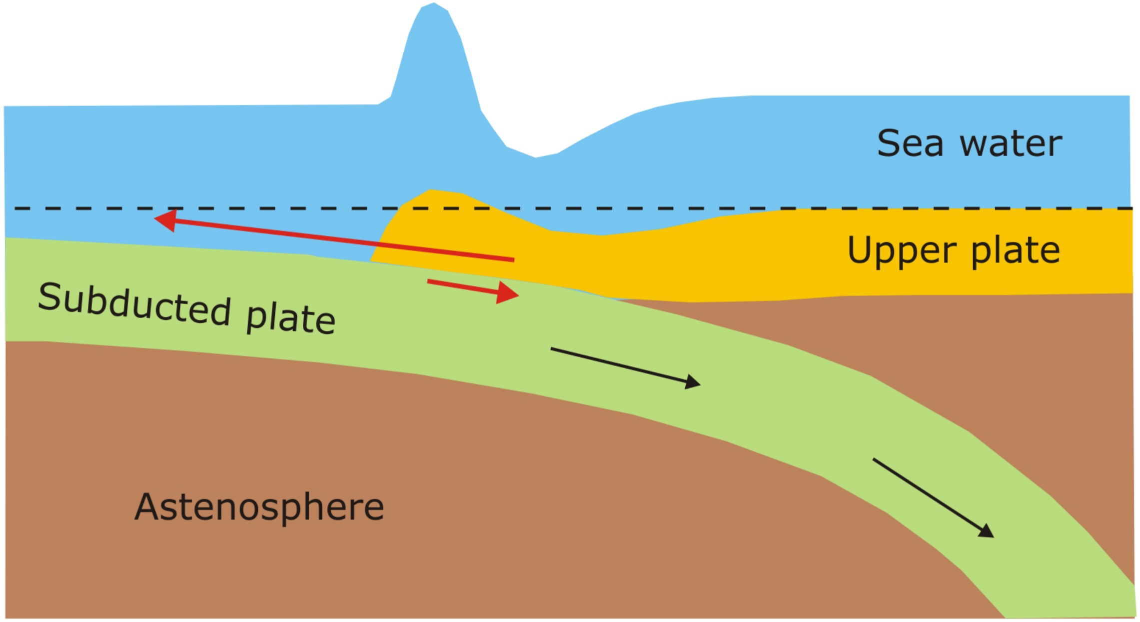
# Mechanism of interplate subduction earthquake



# Mechanism of interplate subduction earthquake

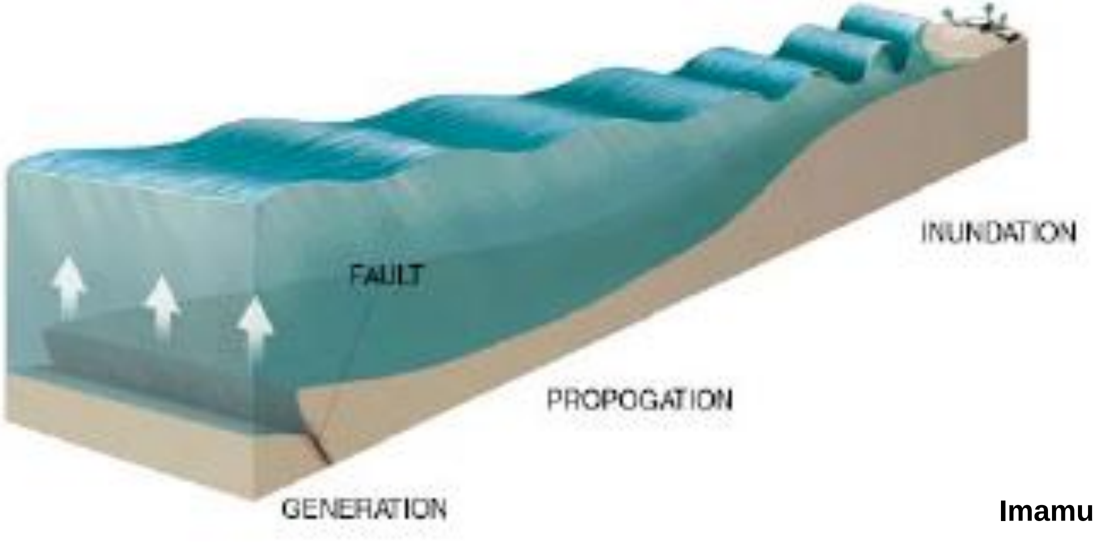


# Mechanism of interplate subduction earthquake



# 3 steps in Tsunami modelling

- Tsunami generation
- Tsunami propagation in deep ocean
- Wave run-up and coastal inundation

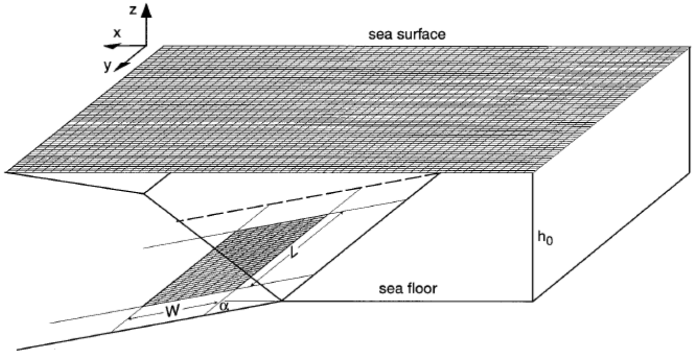


**Imamura (2005)**

# Modeling sea surface displacement

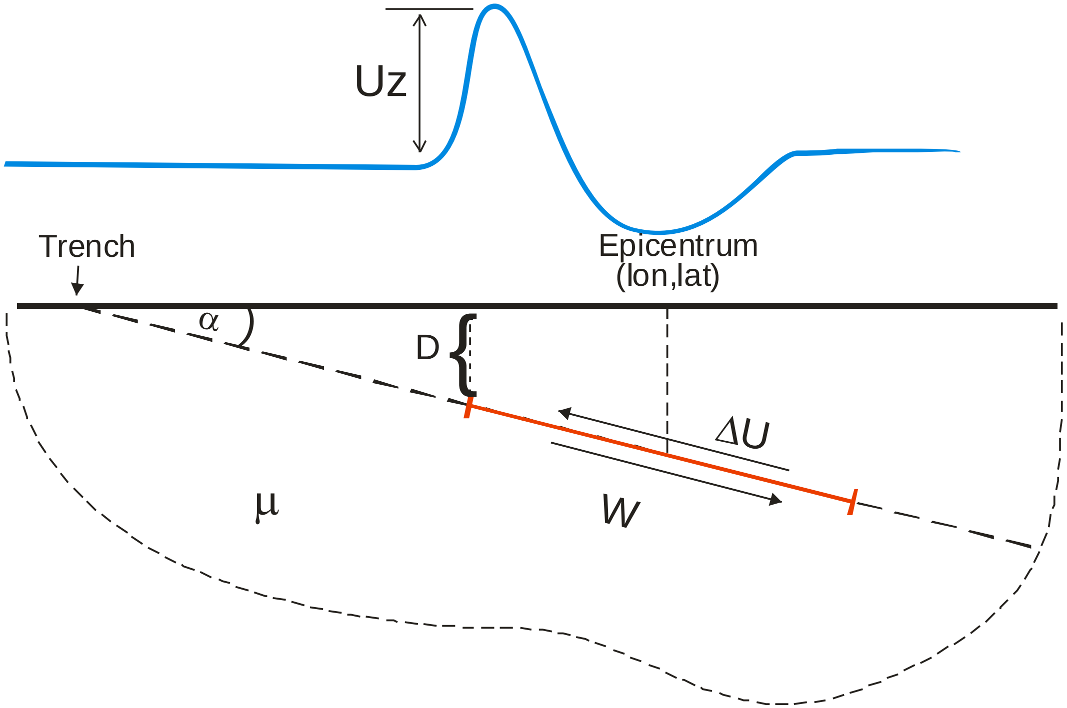
## Rectangular fault

$$M_0 = \mu L W \Delta U$$



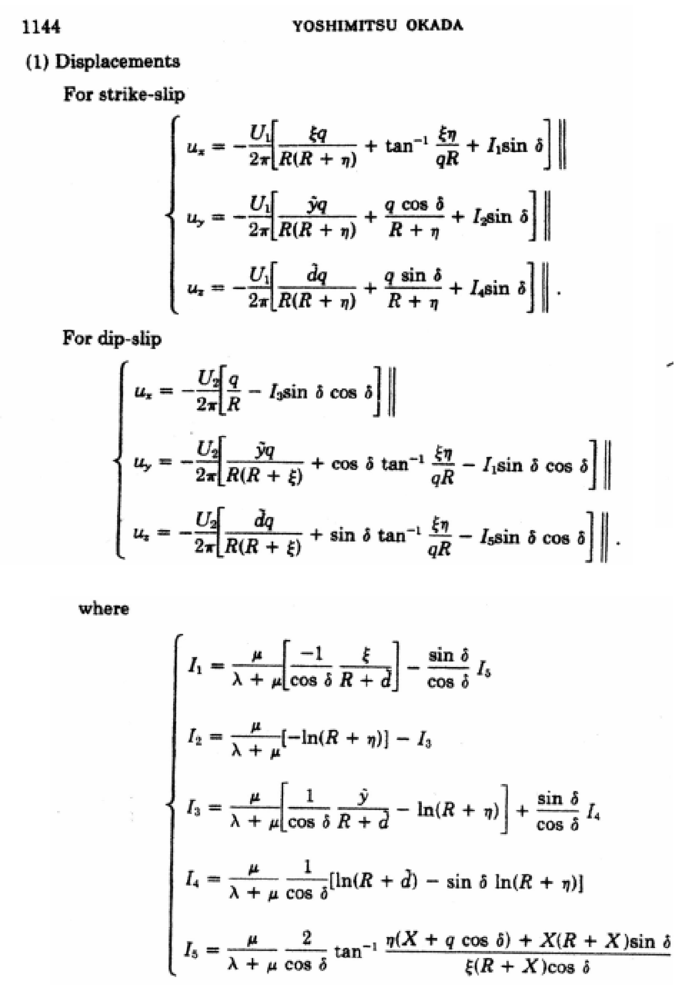
**Model fault parameters:**

Length – *L*
Width – *W*
Dip angle – $\alpha$
Depth – *D*
Slip – $\Delta U$
Rake angle – $\beta$
Shear modulus (rigidity) – $\mu$



# Generation

## Analytical solution for the homogeneous elastic half-space (Okada, 1985)



1144 YOSHIMITSU OKADA

(1) Displacements

For strike-slip

$$\begin{cases} u_x = -\dfrac{U_1}{2\pi}\left[\dfrac{\xi q}{R(R+\eta)} + \tan^{-1}\dfrac{\xi\eta}{qR} + I_1 \sin\delta\right]\Bigg\| \\ u_y = -\dfrac{U_1}{2\pi}\left[\dfrac{\tilde{y}q}{R(R+\eta)} + \dfrac{q\cos\delta}{R+\eta} + I_2 \sin\delta\right]\Bigg\| \\ u_z = -\dfrac{U_1}{2\pi}\left[\dfrac{\tilde{d}q}{R(R+\eta)} + \dfrac{q\sin\delta}{R+\eta} + I_4 \sin\delta\right]\Bigg\| . \end{cases}$$

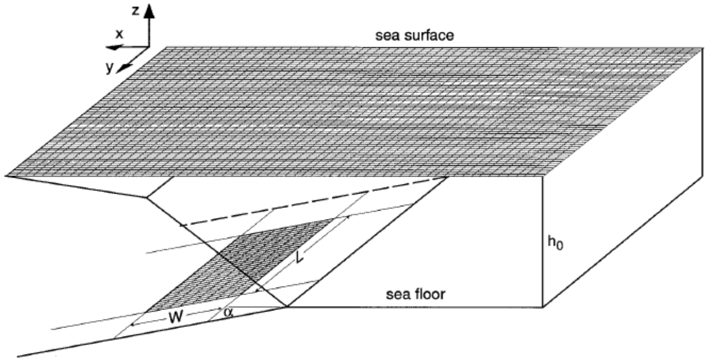
For dip-slip

$$\begin{cases} u_x = -\dfrac{U_2}{2\pi}\left[\dfrac{q}{R} - I_3 \sin\delta\cos\delta\right]\Bigg\| \\ u_y = -\dfrac{U_2}{2\pi}\left[\dfrac{\tilde{y}q}{R(R+\xi)} + \cos\delta\tan^{-1}\dfrac{\xi\eta}{qR} - I_1 \sin\delta\cos\delta\right]\Bigg\| \\ u_z = -\dfrac{U_2}{2\pi}\left[\dfrac{\tilde{d}q}{R(R+\xi)} + \sin\delta\tan^{-1}\dfrac{\xi\eta}{qR} - I_5 \sin\delta\cos\delta\right]\Bigg\| . \end{cases}$$

where

$$\begin{cases} I_1 = \dfrac{\mu}{\lambda+\mu}\left[\dfrac{-1}{\cos\delta}\dfrac{\xi}{R+\tilde{d}}\right] - \dfrac{\sin\delta}{\cos\delta} I_5 \\ I_2 = \dfrac{\mu}{\lambda+\mu}[-\ln(R+\eta)] - I_3 \\ I_3 = \dfrac{\mu}{\lambda+\mu}\left[\dfrac{1}{\cos\delta}\dfrac{\tilde{y}}{R+\tilde{d}} - \ln(R+\eta)\right] + \dfrac{\sin\delta}{\cos\delta} I_4 \\ I_4 = \dfrac{\mu}{\lambda+\mu}\dfrac{1}{\cos\delta}[\ln(R+\tilde{d}) - \sin\delta\ln(R+\eta)] \\ I_5 = \dfrac{\mu}{\lambda+\mu}\dfrac{2}{\cos\delta}\tan^{-1}\dfrac{\eta(X+q\cos\delta)+X(R+X)\sin\delta}{\xi(R+X)\cos\delta} \end{cases}$$

**Analytical expressions for *$U_i(x,y)$, $i=(x,y,z)$***



**Model parameters:**

1) Fault geometry
$L, W, \alpha, D_{,}, \Delta U, \beta$

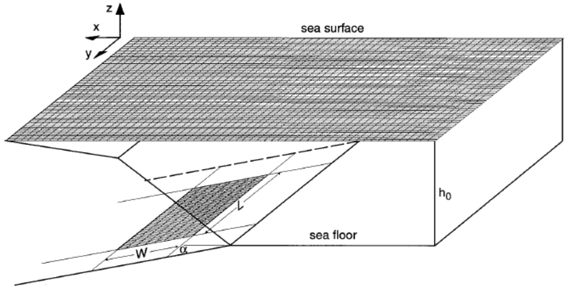
2) Position of obs. point
x,y

3) Parameters of the media
$\lambda, \mu$

## Scaling laws

Empirically calibrated relations between earthquakes parameters

We would like to know 3 fault parameters: length ($L$), width ($W$) and slip ($\Delta U$) from the seismic moment ($M_0$) only.



$$M_0 = \mu \, L \, W \, \Delta U$$

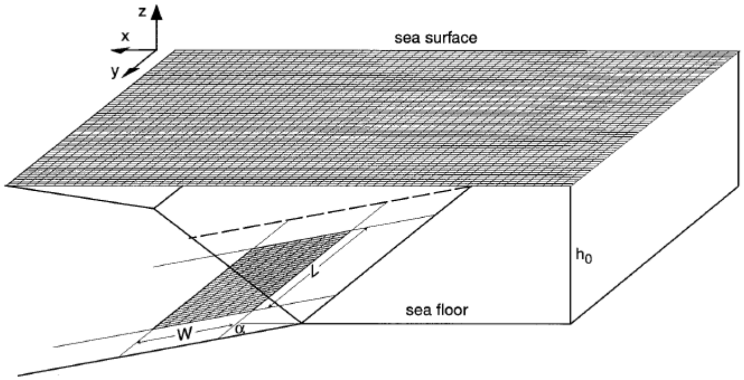
If, e.g., $L = L(M_0)$ and $W = W(M_0)$,

then: $M_0 = \mu \; L(M_0) \; W(M_0) \; \Delta U$

and: $\Delta U = M_0 / ( \mu \; L(M_0) \; W(M_0) )$

# Generation

$$M_0 = \mu L W \Delta U$$



**Measured:**

$M_0$, *location*

**Finite fault parameters that we do need:**

| | |
|---|---|
| Length – $L$ | $M_0$ + scaling laws |
| Width – $W$ | $M_0$ + scaling laws |
| Dip angle – $\alpha$ | subduction zone geometry |
| Depth – $D$ | location + subduction zone geometry |
| Slip – $\Delta U$ | $M_0$ + scaling laws |
| Rake angle – $\beta$ | assumed |
| Shear modulus (rigidity) – $\mu$ | material constant |

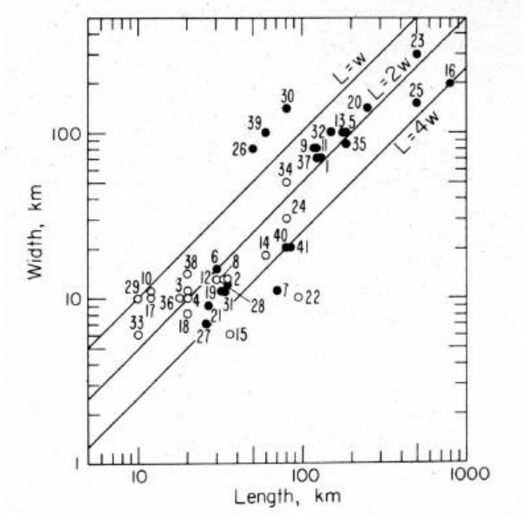
$\mu$: crustal rocks – 30-40 GPa, dry olivine – 74 GPa;

$M_0 = \mu L W \Delta U$

# Scaling laws: physical reasoning

- Rupture can grow in all directions.
  Hence: $L \sim W$
- Max. strain $\varepsilon \sim \Delta U/W$ is related to rock *strength.*
  Hence: $\Delta U \sim W$
- Thus one might expect that: $M_0 \sim L^3$

Geller (1976)



But, for large earthquakes these relations should break down!

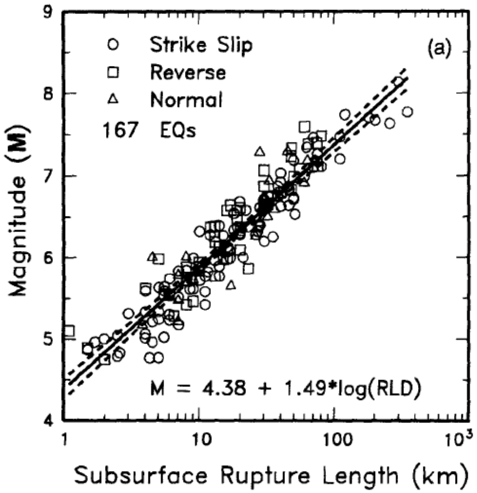
- $W$ cannot grow as far as $L$ can do: temperature increases with depth – material is no more brittle
- Similarly, $\Delta U$ may stop growing to keep strain
- Hence we might expect that for large events: $M_0 \sim L$

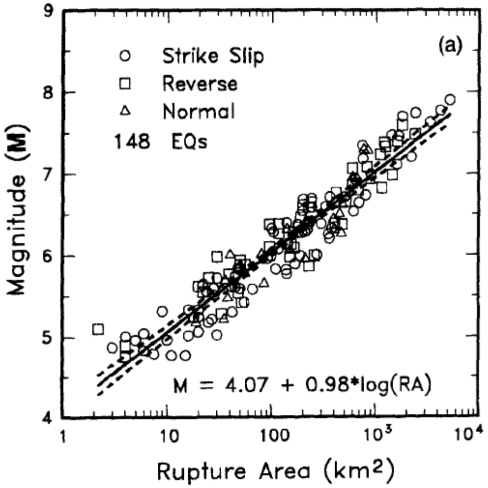
# Generation

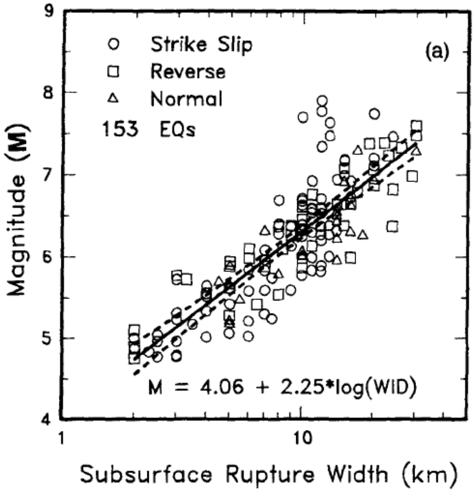
$M_0 = \mu L W \Delta U$

## Scaling laws

### Calibrations of Wells and Coppersmith (1994)







Wells, D.L., Coppersmith, K.J. (1994). New empirical relationships among magnitude, rupture length, rupture width, rupture area, and surface displacement. Bull. Seism. Soc. Am. 84, 974-1002.

# Quick fault model: an example

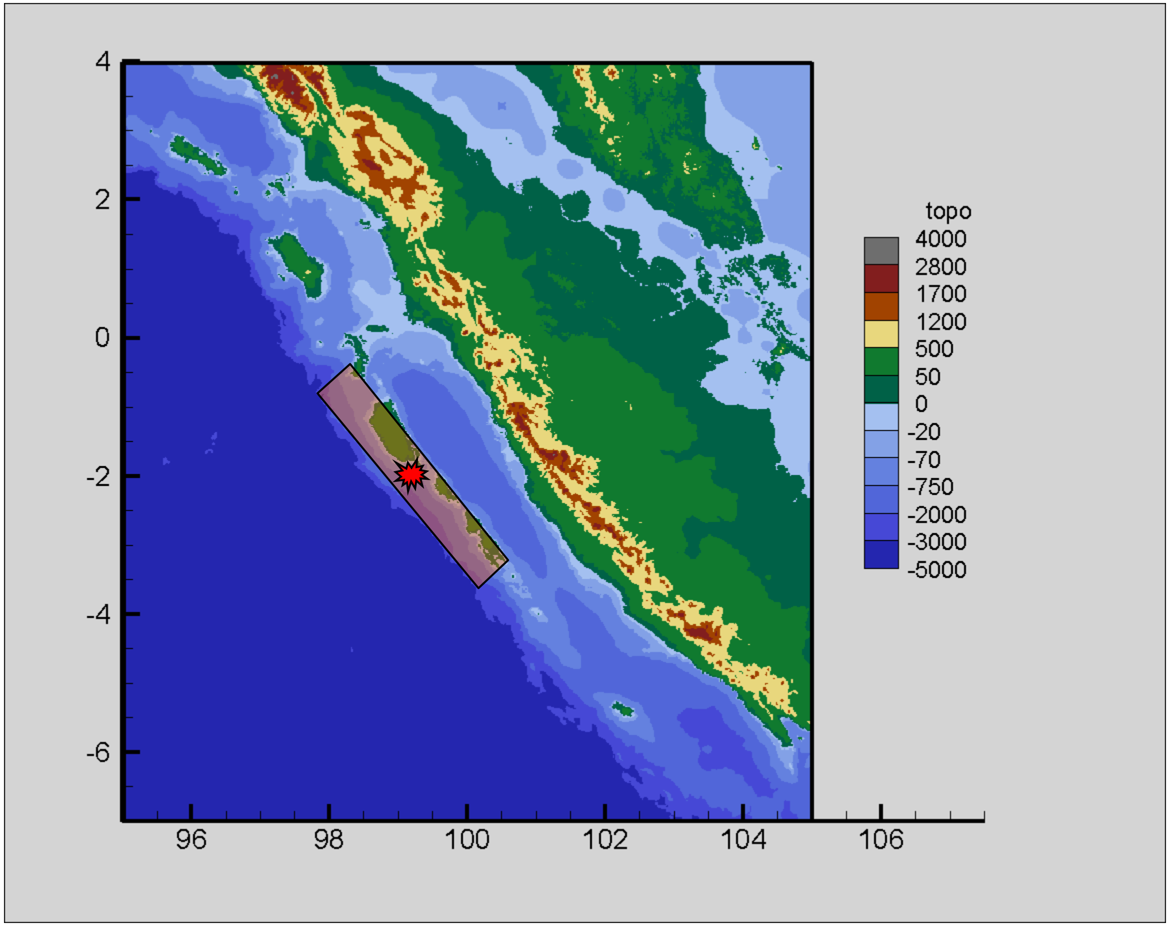
Event with $M_w = 8.5$

$L$ and $W$ from scaling laws:

$L \sim 375$ km
$W \sim 50$ km

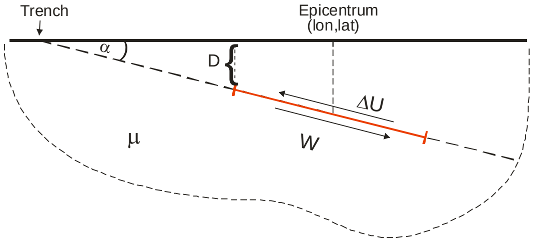
Slip:
$\Delta U = M_0 / (\mu L W)$

$\Delta U \sim 13$ m

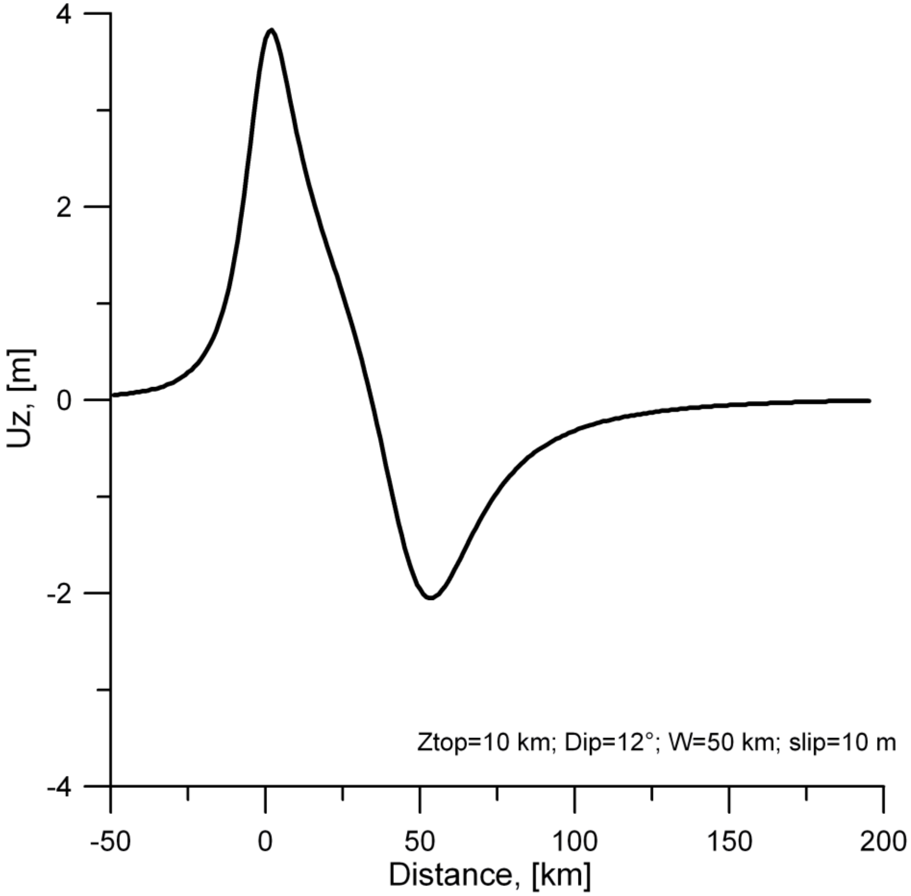


$$M_0 = \mu L W \Delta U$$

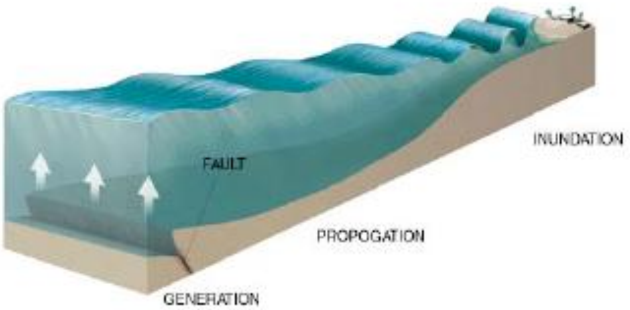
# Vertical displacement
# 1D- perspective



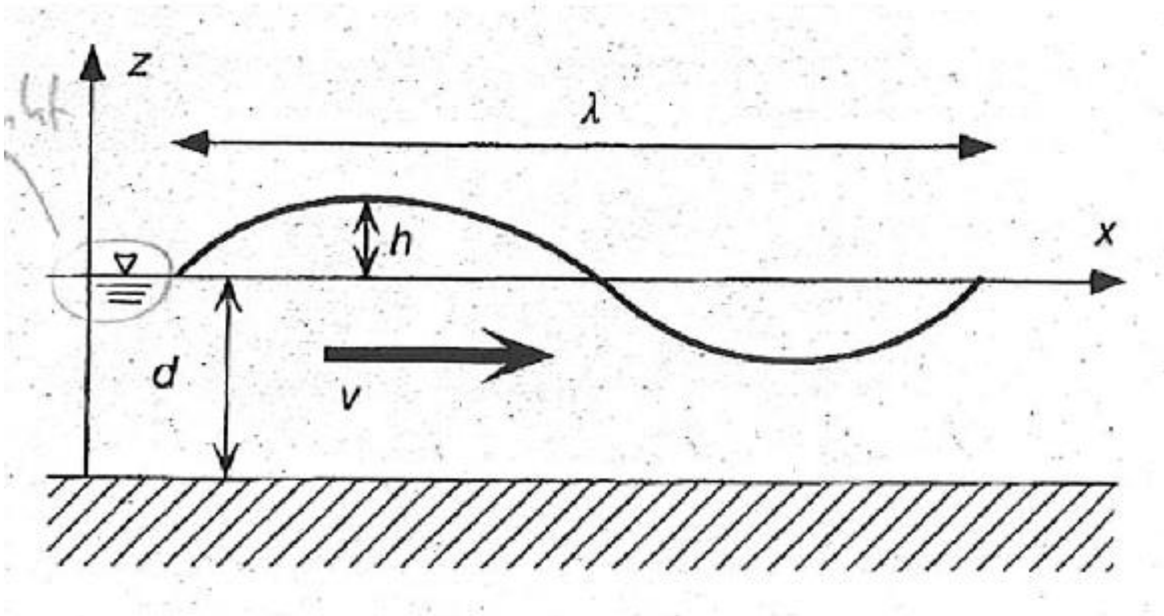
Remark: here and below Ztop=D



# Propagation



## Modeling: Nomenclature



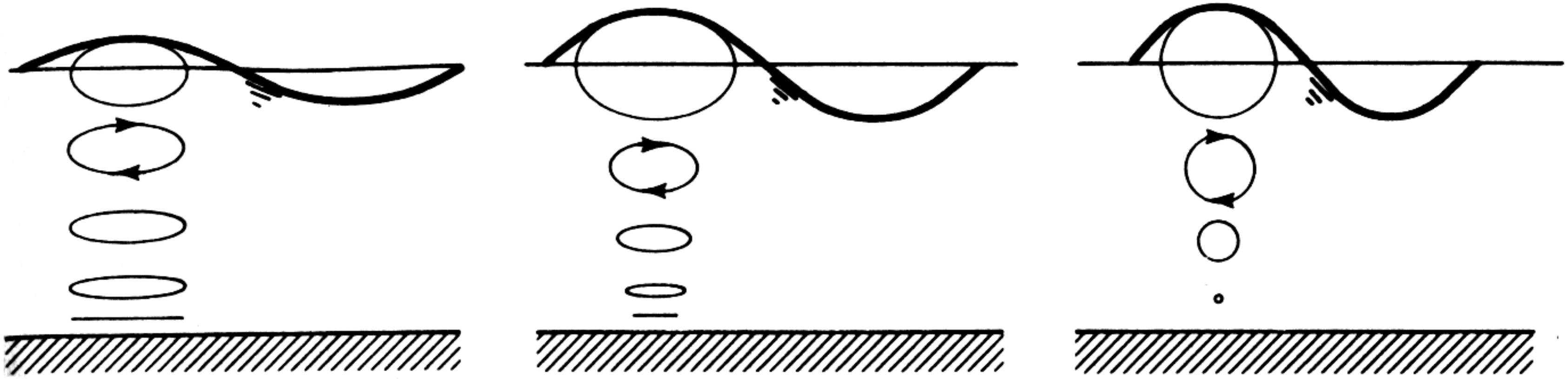
# Propagation

**Full 3D model:**

$$\rho \frac{\partial \mathbf{u}}{dt} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \eta \nabla^2 \mathbf{u}$$ - 3D Navier-Stokes equation

$$\nabla \mathbf{u} = 0$$ - mass conservation (incompressible)

# Tsunami is a long wave !



Long-wave

$V_x \neq f(z)$

$(\frac{h}{L} < \frac{1}{20})$

Short-wave

$V_x = f(z)$

$(\frac{h}{L} > \frac{1}{2})$

## Modeling: Governing equations

The simplest form: Linear shallow water (long wave) equations

$$\frac{\partial u}{\partial x}d+\frac{\partial h}{\partial t}=0$$ -- balance of mass

$$\frac{\partial u}{\partial t}+g\frac{\partial h}{\partial x}=0$$ -- balance of momentum

Note: Solution (***h*** and ***u***) depends on one single variable: water depth ***d*** !

# Propagation

## Modeling: Governing equations

$$\frac{\partial u}{\partial x} d + \frac{\partial h}{\partial t} = 0 \qquad \Bigg| \; \frac{\partial}{\partial t}$$

$$\frac{\partial u}{\partial t} + g \frac{\partial h}{\partial x} = 0 \qquad \Bigg| \; \frac{\partial}{\partial x}$$

$$\frac{\partial^2 u}{\partial t \partial x} d + \frac{\partial^2 h}{\partial t^2} = 0$$

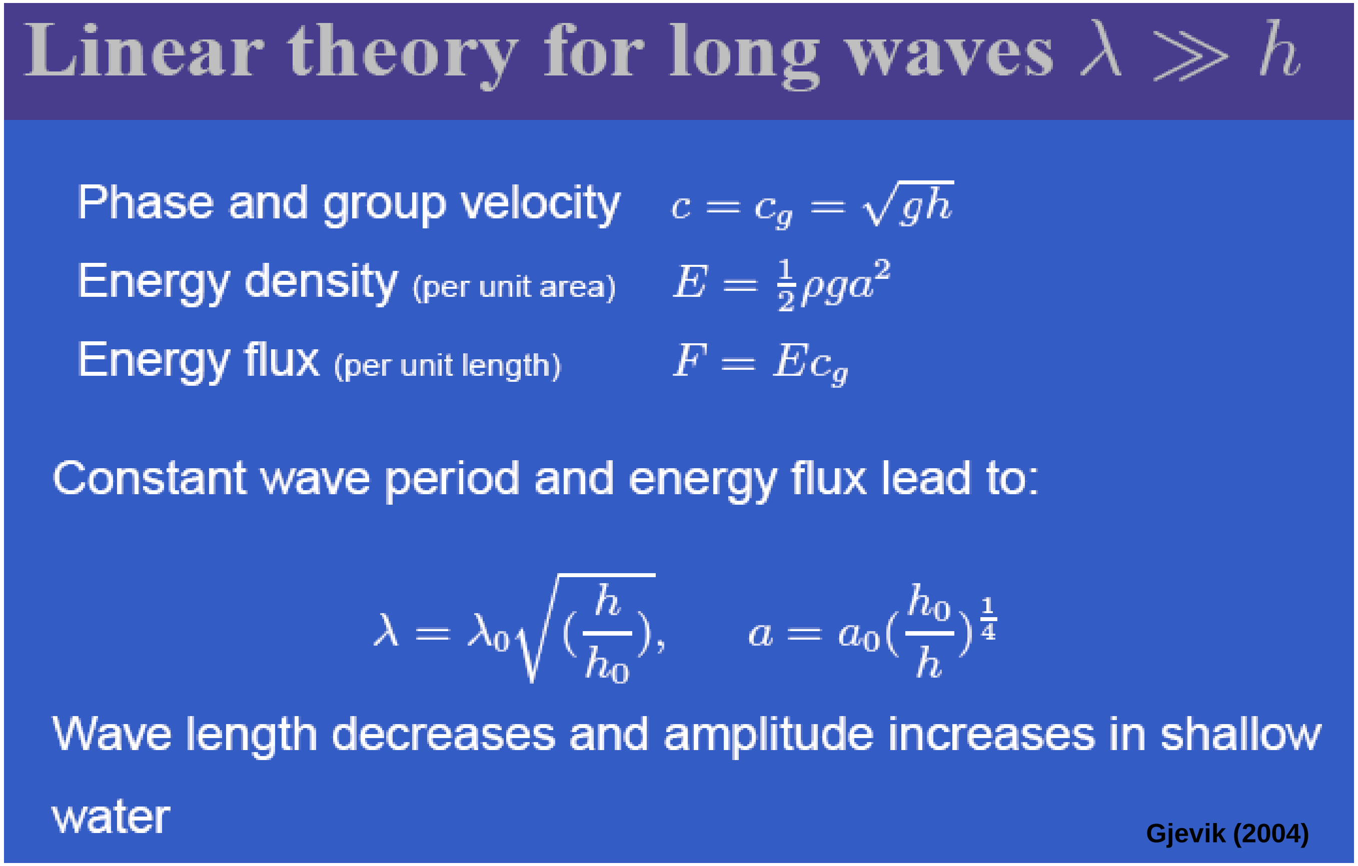
$$\frac{\partial^2 u}{\partial t \partial x} + g \frac{\partial^2 h}{\partial x^2} = 0 \quad \Rightarrow \quad \boxed{\frac{\partial^2 h}{\partial t^2} = gd \frac{\partial^2 h}{\partial x^2}}$$

-- wave equation

$$\boxed{c = \sqrt{gd}}$$

-- wave speed

# Propagation



## Linear theory for long waves $\lambda \gg h$

Phase and group velocity $c = c_g = \sqrt{gh}$

Energy density (per unit area) $E = \frac{1}{2}\rho g a^2$

Energy flux (per unit length) $F = E c_g$

Constant wave period and energy flux lead to:

$$\lambda = \lambda_0 \sqrt{\left(\frac{h}{h_0}\right)}, \qquad a = a_0 \left(\frac{h_0}{h}\right)^{\frac{1}{4}}$$

Wave length decreases and amplitude increases in shallow water

**Gjevik (2004)**

# Tsunami wave characteristics

# Shallow-water equations: Numerical aspects

Numerical schemes:

(1) Finite differences on structural grids

*Pro*: easy to implement, robust, easy grids, quick, straightforward parallelization of computations

*Contra*: constant resolution, need for nested grids in coastal regions

*Examples*: TUNAMI-family, MOST, FUNWAVE

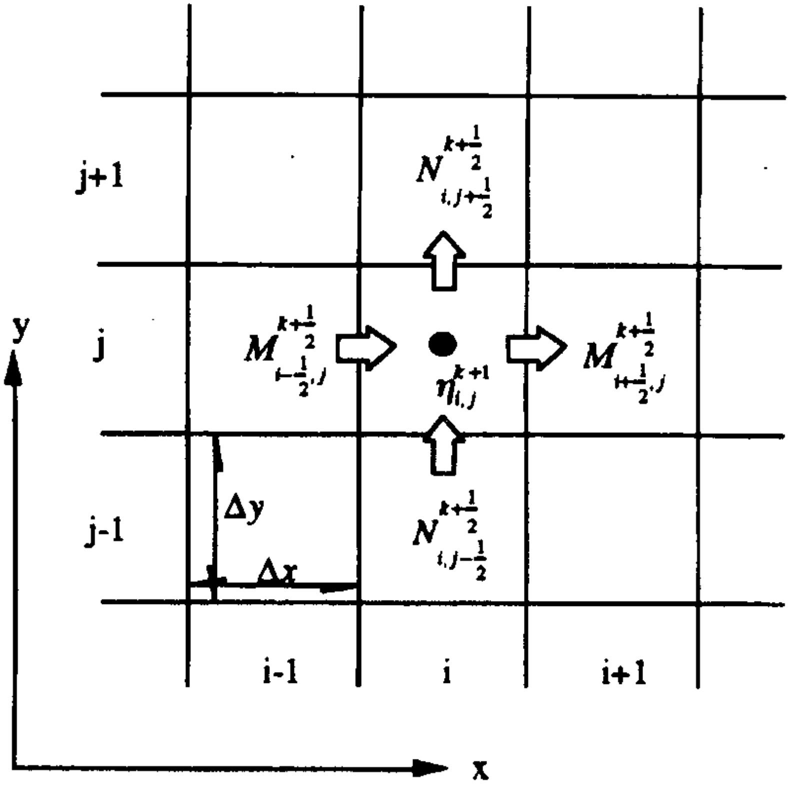
(2) Finite elements on unstructural grids

*Pro*: single computational domain for deep-ocean propagation and inundation

*Contra*: time consuming, stability problems, hard to program, complex grids

*Examples*: TsunAwi, ANUGA, Uni Bologne

# Finite differences on structural grids



**Imamura (1996)**

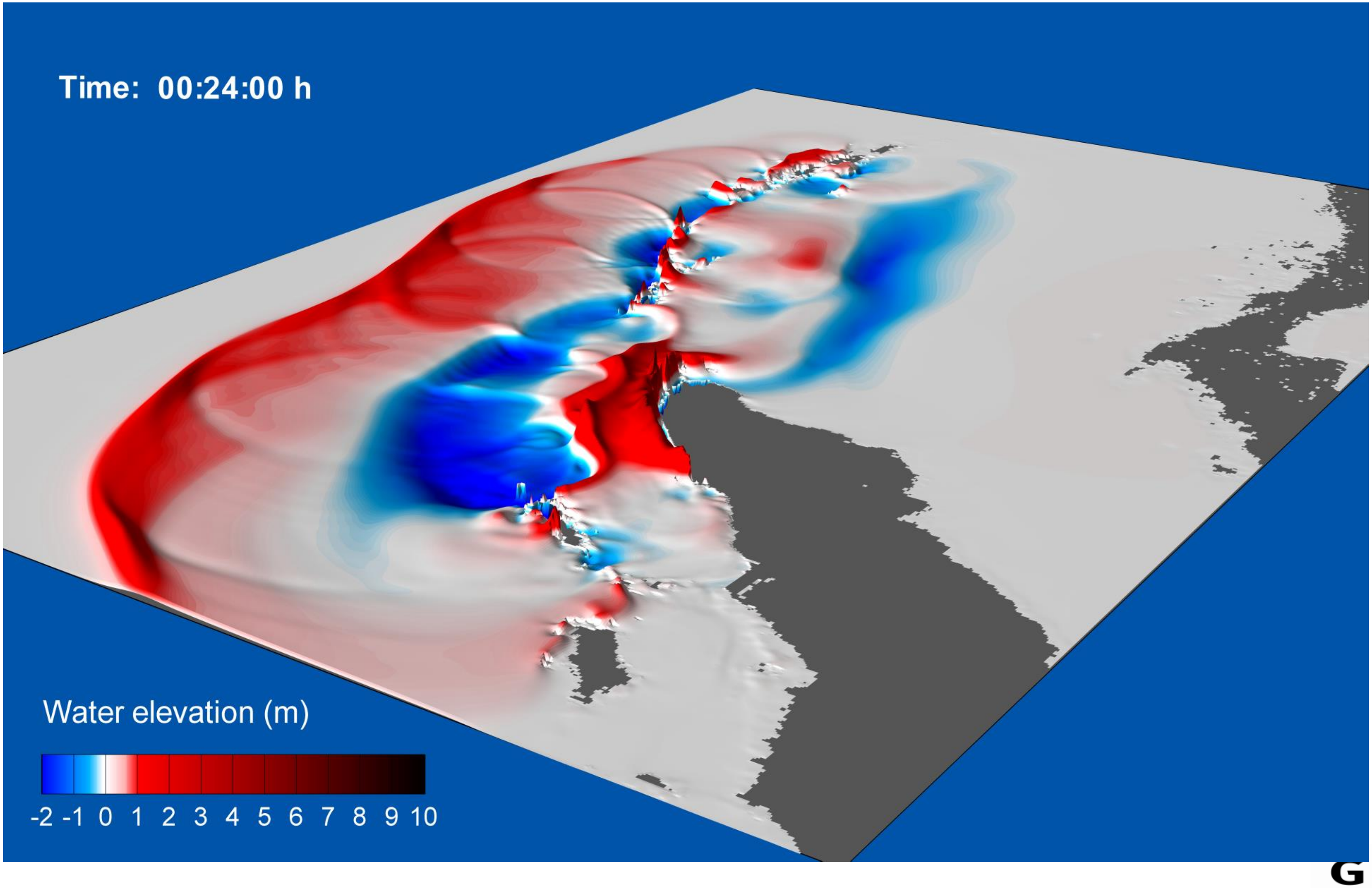
$$\frac{\partial\eta}{\partial t} = \frac{1}{\Delta t}\left[\eta^{k+1}_{i,j} - \eta^{k}_{i,j}\right]$$

$$\frac{\partial M}{\partial x} = \frac{1}{\Delta x}\left[M^{k+\frac{1}{2}}_{i+\frac{1}{2},j} - M^{k+\frac{1}{2}}_{i-\frac{1}{2},j}\right]$$

$$\frac{\partial N}{\partial y} = \frac{1}{\Delta y}\left[N^{k+\frac{1}{2}}_{i,j+\frac{1}{2}} - N^{k+\frac{1}{2}}_{i,j-\frac{1}{2}}\right]$$

# Propagation

24 Dec 2004



# Nested grids to increase resolution in critical places

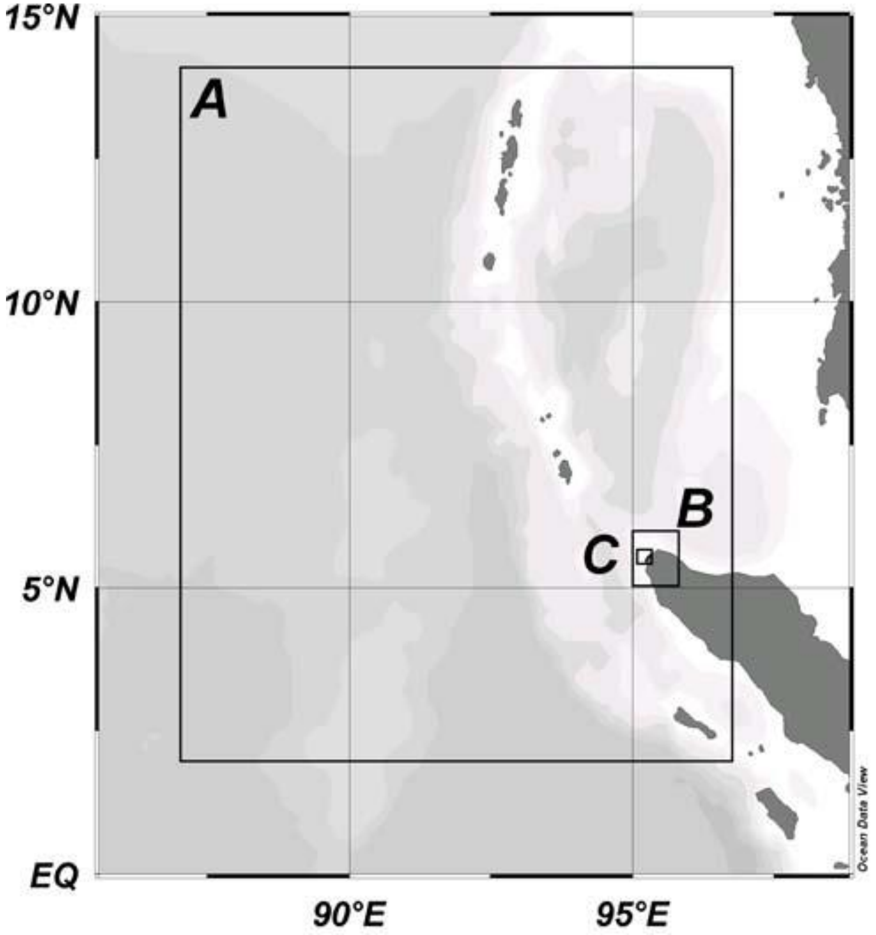
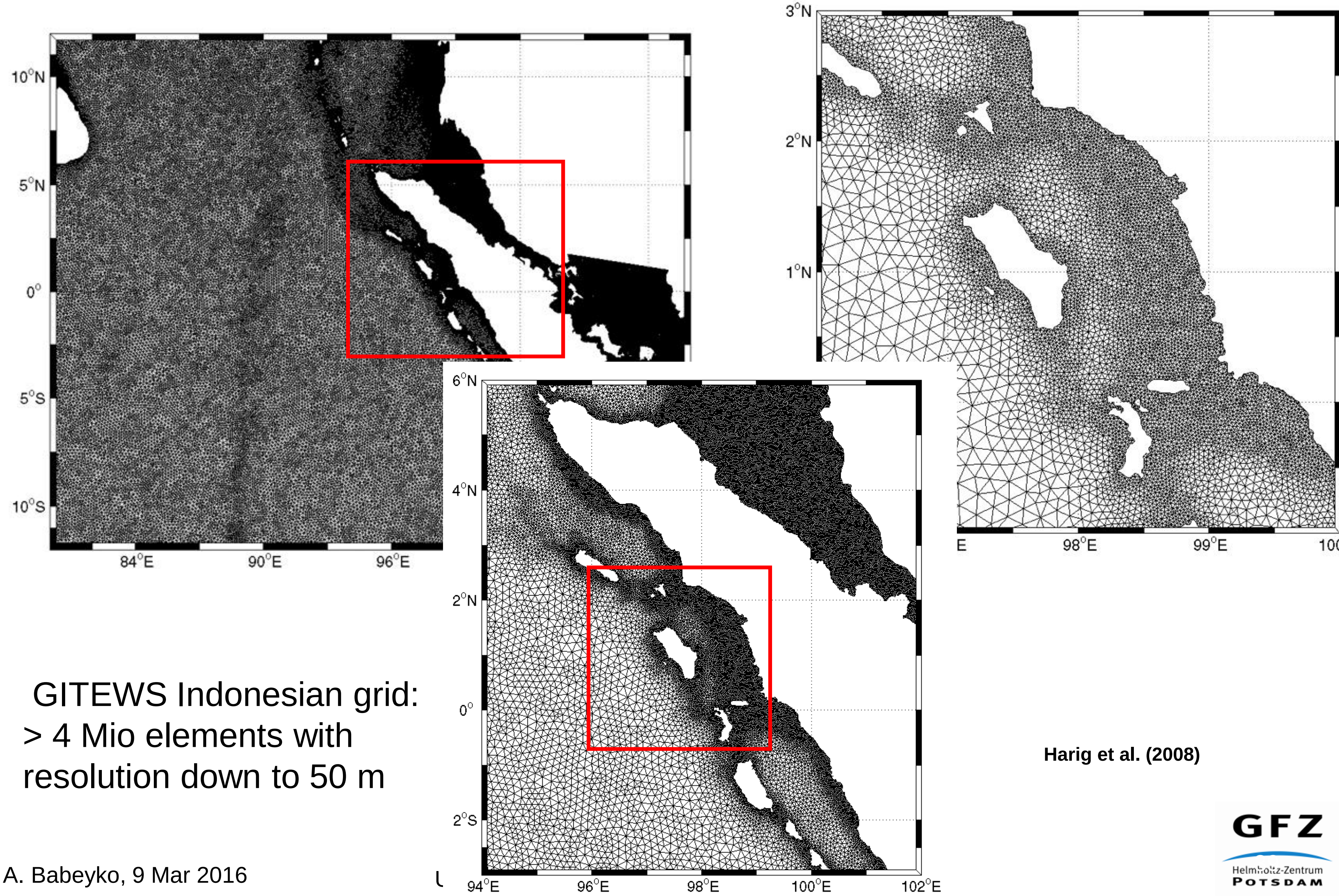


**Fig. 2** The *upper panel* shows the nested grids *A*, *B*, and *C* in TUNAMI (*grid A*: 1,280 × 1,354 nodes, dx = 900 m; *grid B*: 297 × 357 nodes, dx = 300 m; *grid C*: 289 × 274 nodes, dx = 100 m).

**Harig et al. (2008)**

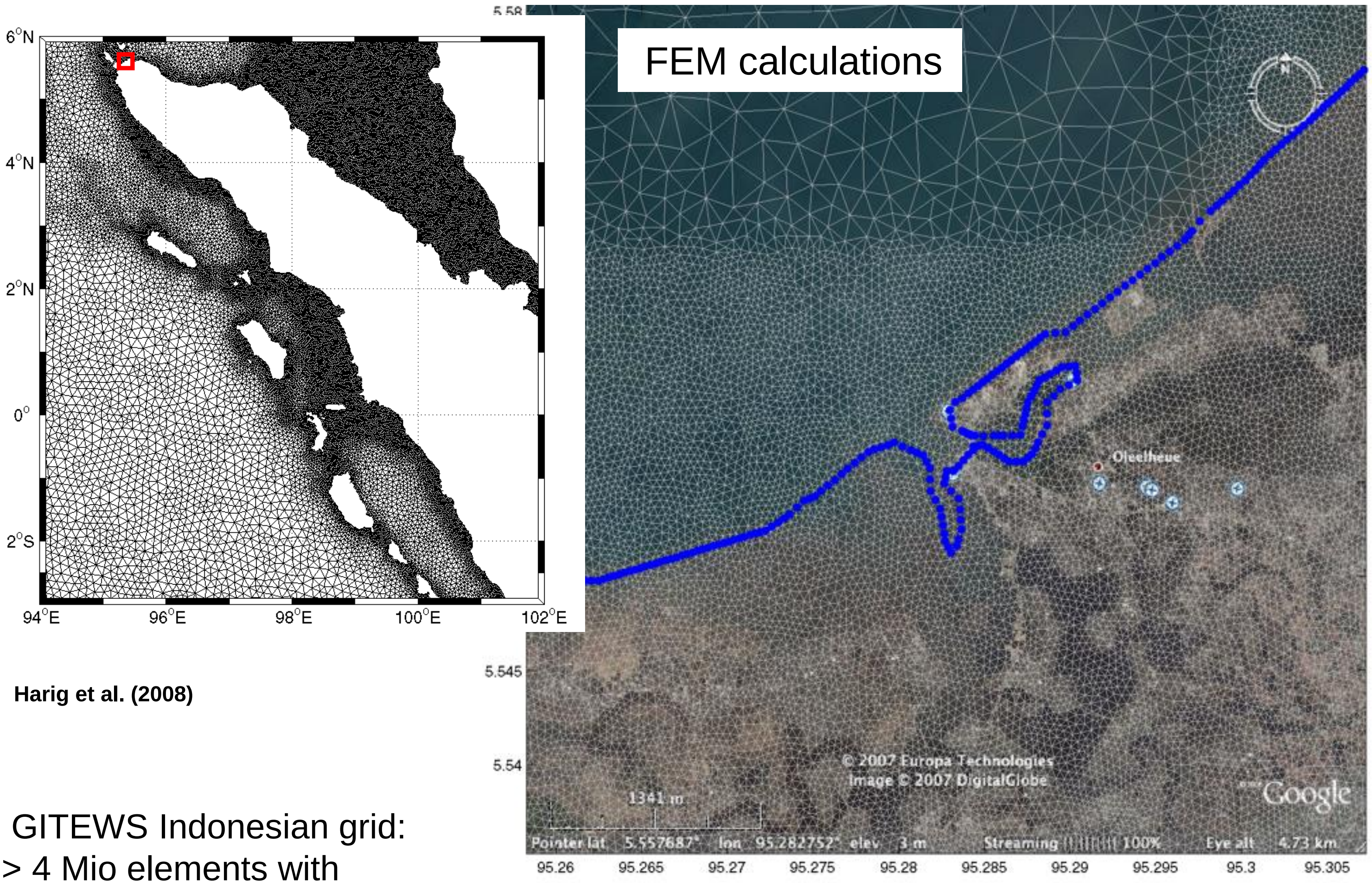
# FEM calculations



GITEWS Indonesian grid:
> 4 Mio elements with
resolution down to 50 m

Harig et al. (2008)

# Propagation



**Harig et al. (2008)**

GITEWS Indonesian grid:
> 4 Mio elements with
resolution down to 50 m

## Comments on simulation of wave run-up and coastal inundation

- Most demanding to data resolution and accuracy as well as to computational cost
  - Global bathymetry and topography datasets not enough
  - Grid resolution ~10 m: tens of million of nodes
- High-resolution local data on topography often not available
- Approximations from deep water (~ 50-100 m depth) is commonly used
  - Green's law: $h_2 = h_1*(d_2/d_1)^{1/4}$
  - Use precomputed 1D characteristic profiles

## Numerical Modeling for:

1. Tsunami early warning
2. Tsunami hazard assessment
3. Integrative testing of the TEWS
4. Personnel teaching and training

## Numerical Modeling for:

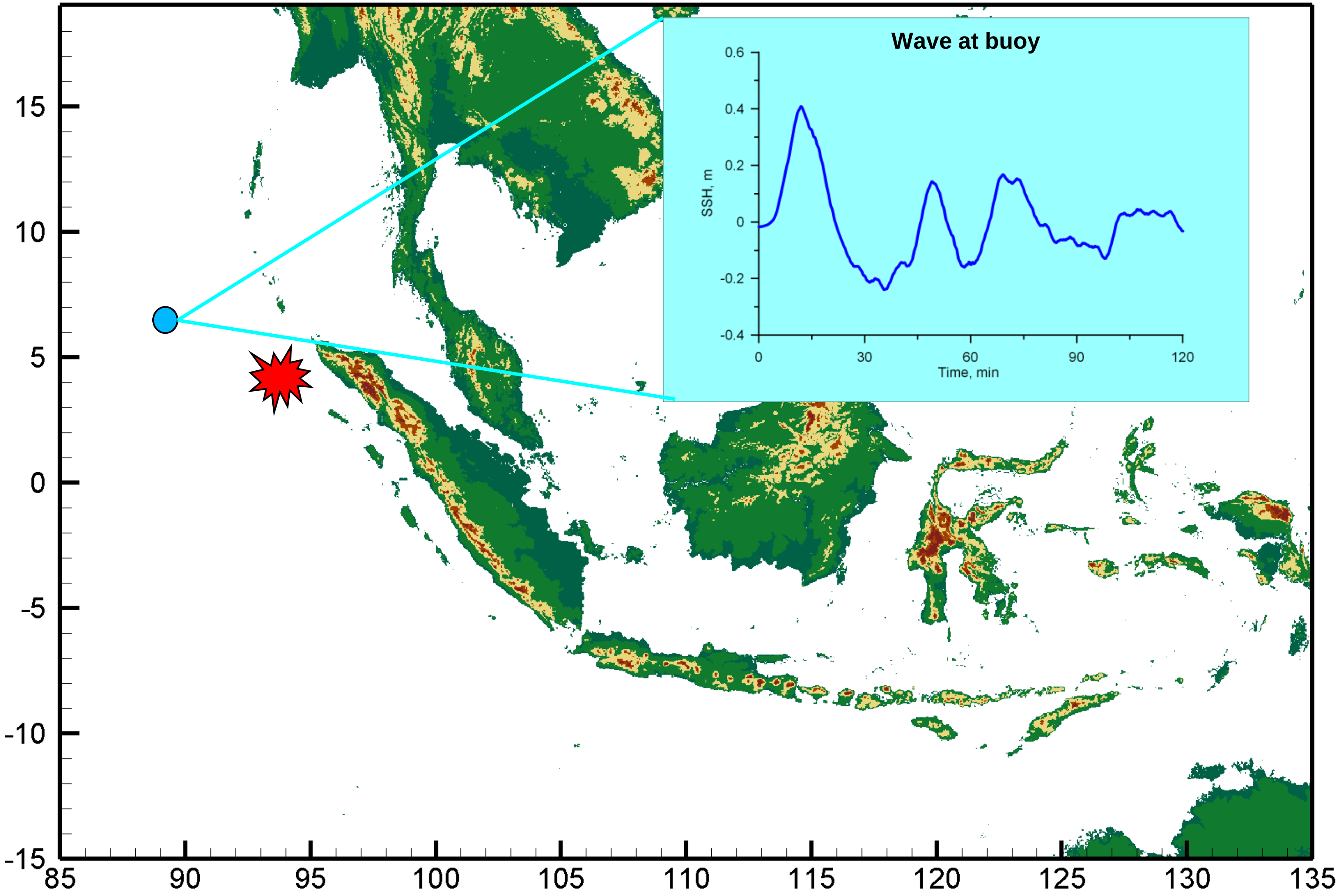
1. Tsunami early warning
2. Tsunami hazard assessment
3. Integrative testing of the TEWS
4. Personnel teaching and training

**Modeling for Tsunami Early Warning …**

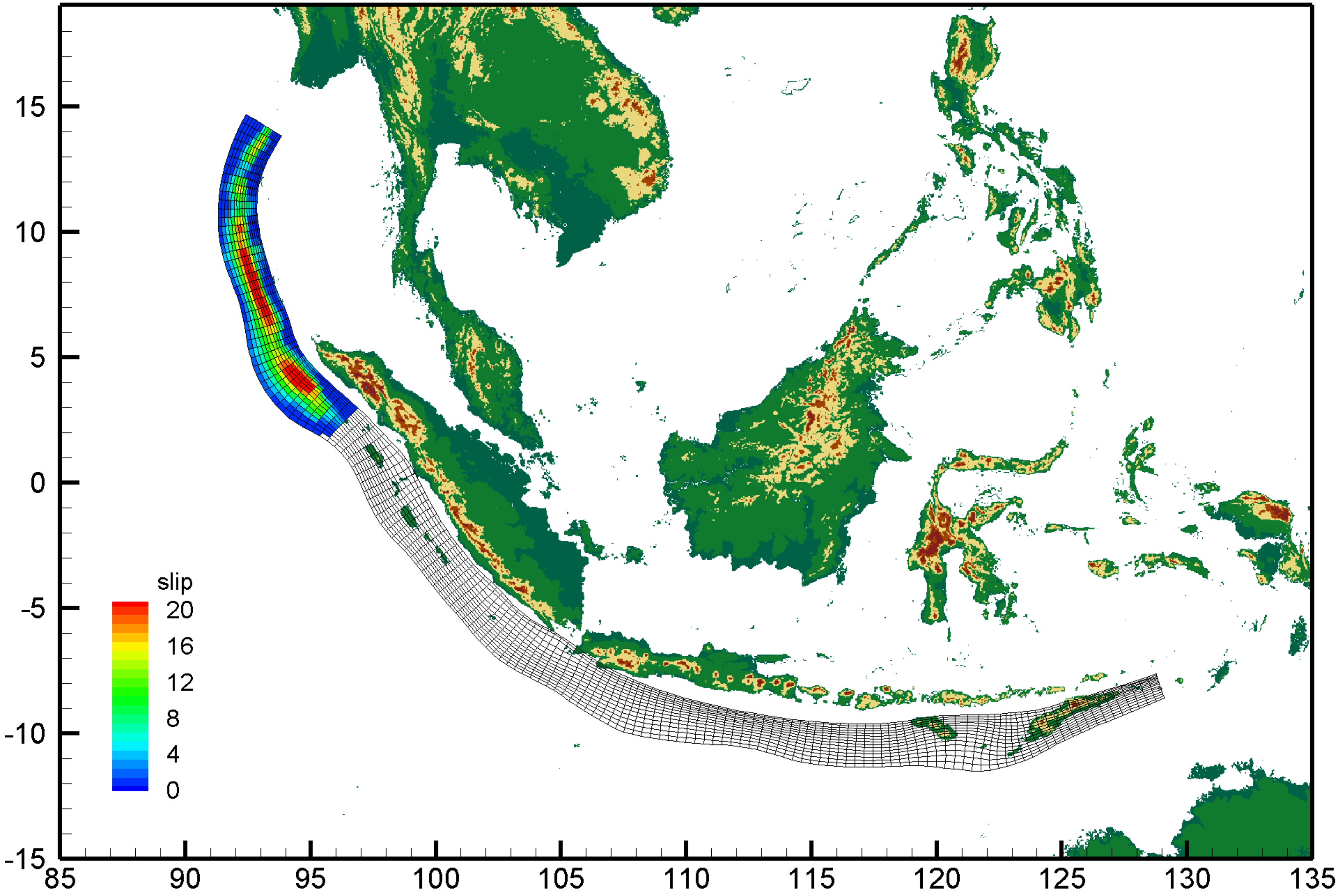
… to understand a physical process.

To get a consistent picture of what has happened, what is still going and what is expected to be …

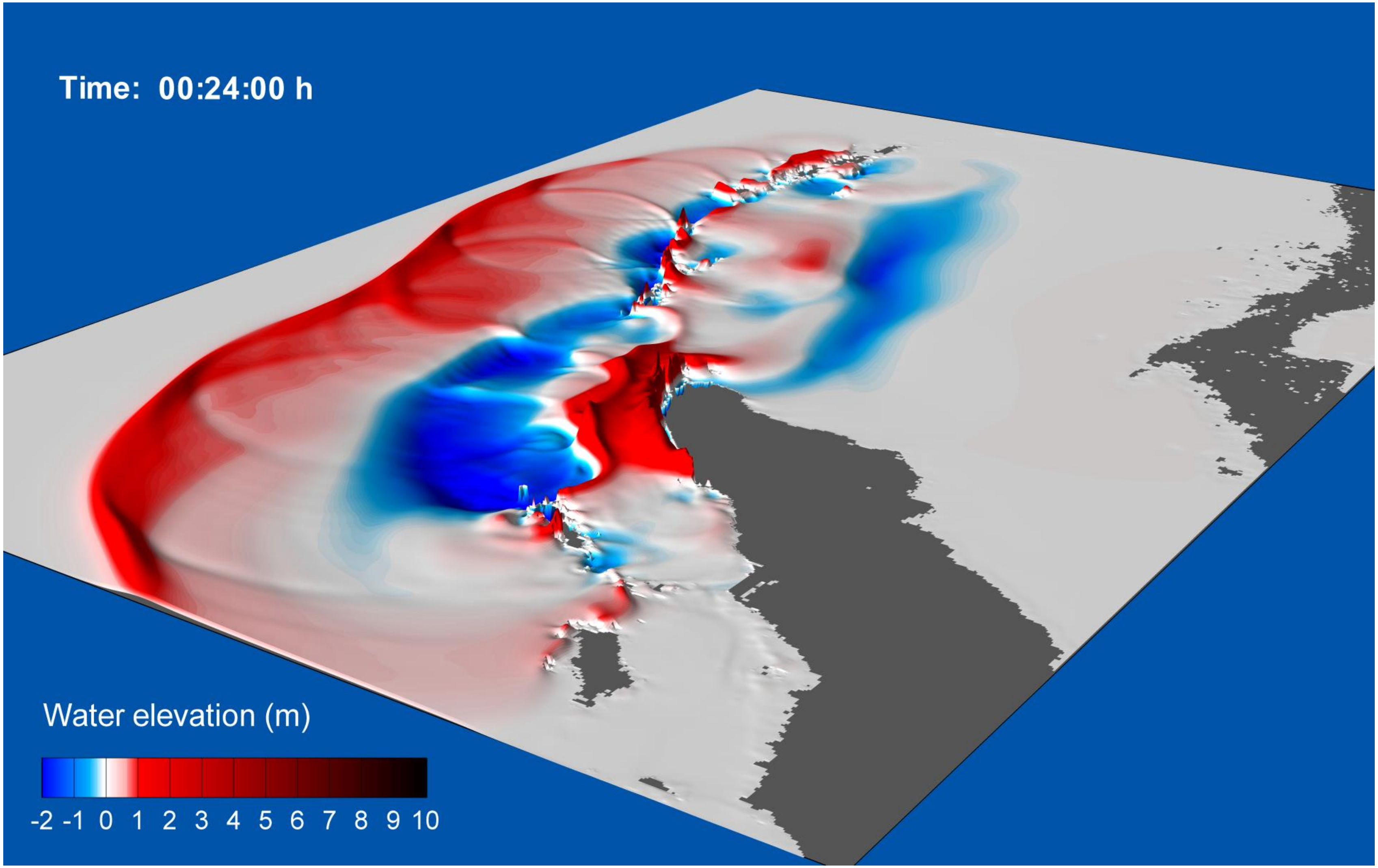
# Modeling for early warning



# Modeling for early warning



# Modeling for early warning



# Tsunami Early Warning Systems: Principles of Operation

**Far-field (PTWC, India, Australia,…):**
there is some time to take decision
more tolerant to the source: important are Mw and orientation

**Near-field (Japan, Indonesia, Chile,…):**
extremely short warning times
needs source position, geometry and slip distribution

# Far-field: Pacific Tsunami Warning Center

New: **SIFT** – **S**hort-term **I**nundation **F**orecasting for **T**sunamis

based on concept of unit sources and their Green's functions

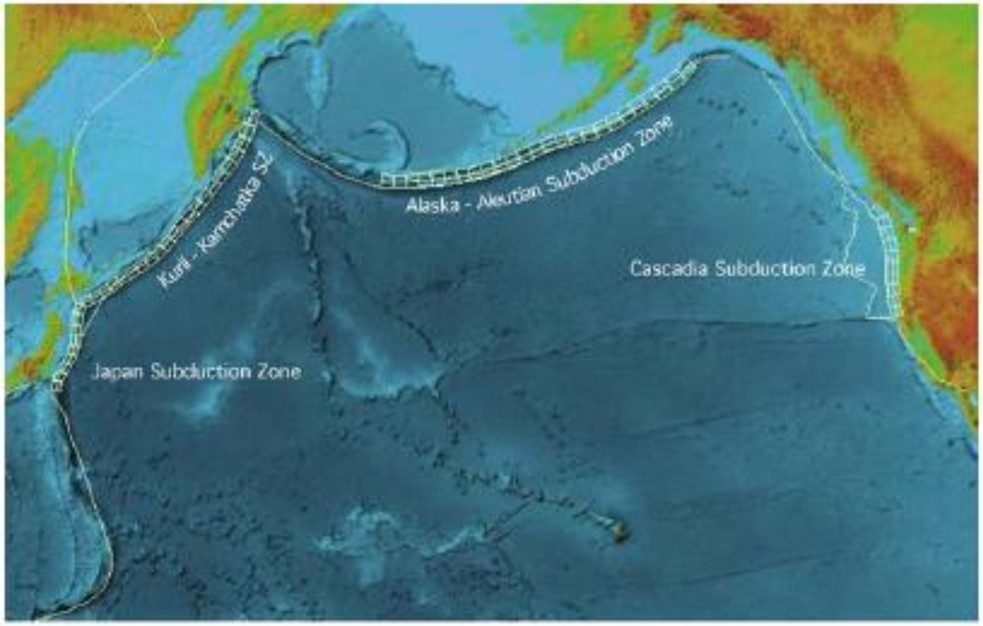
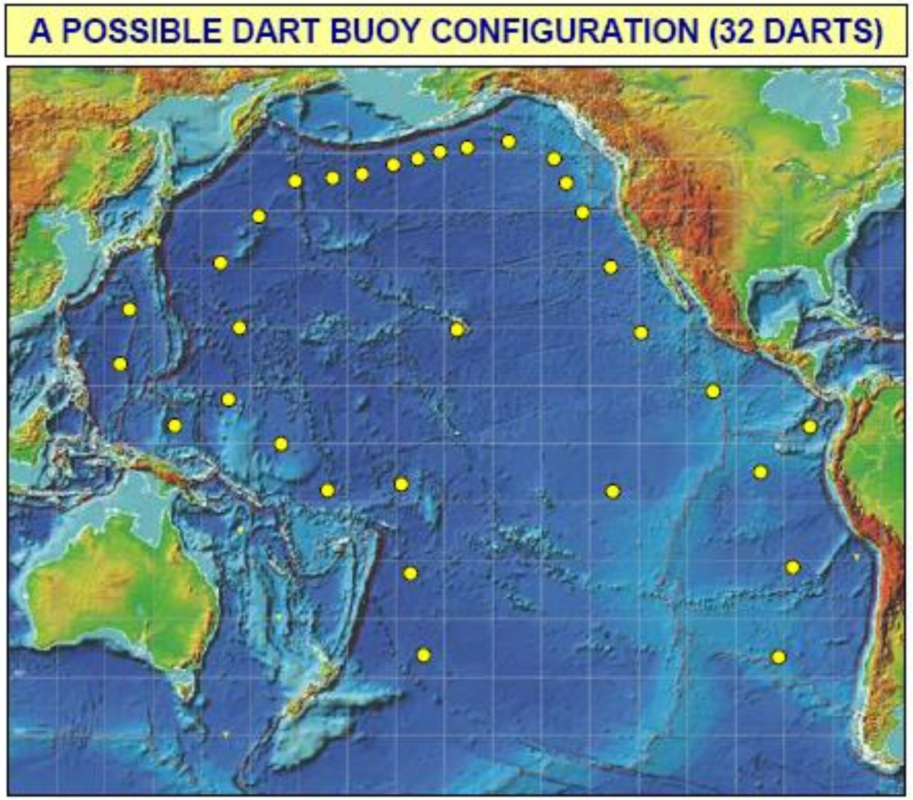


*Figure 5.* North Pacific details of the Pacific-wide forecast model database. Bathymetric data for the database computation is shown as a shaded relief map. White rectangles show fault planes for the unit sources included in the database. Major plate boundaries are shown as white lines.

**Titov et al. (2005)**

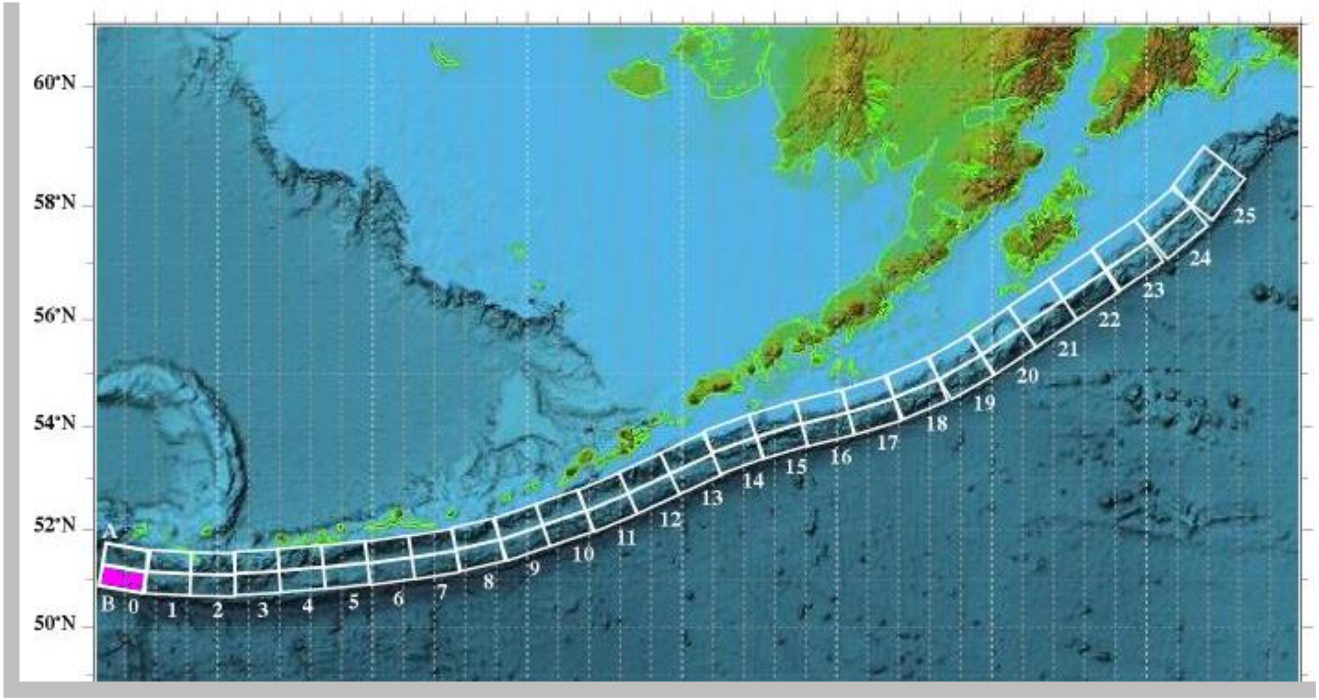
New: **SIFT** – **S**hort-term **I**nundation **F**orecasting for **T**sunamis



**from presentation of McCreery (2005)**

# New: **SIFT** – **S**hort-term **I**nundation **F**orecasting for **T**sunamis

- Each ‘fault box’ (labeled a0..an, b0…bn below ) is an independent source location. One ~7 GByte dataset is computed for each box, using an Mw 7.5 , 1 m slip seismic source. This defines a ‘unit source’.
- A weighted combination of stored data from several sources produces the final wave prediction we will use during an event.
- The ‘best’ combination is obtained by matching prediction to real time DART data. That’s the inversion step



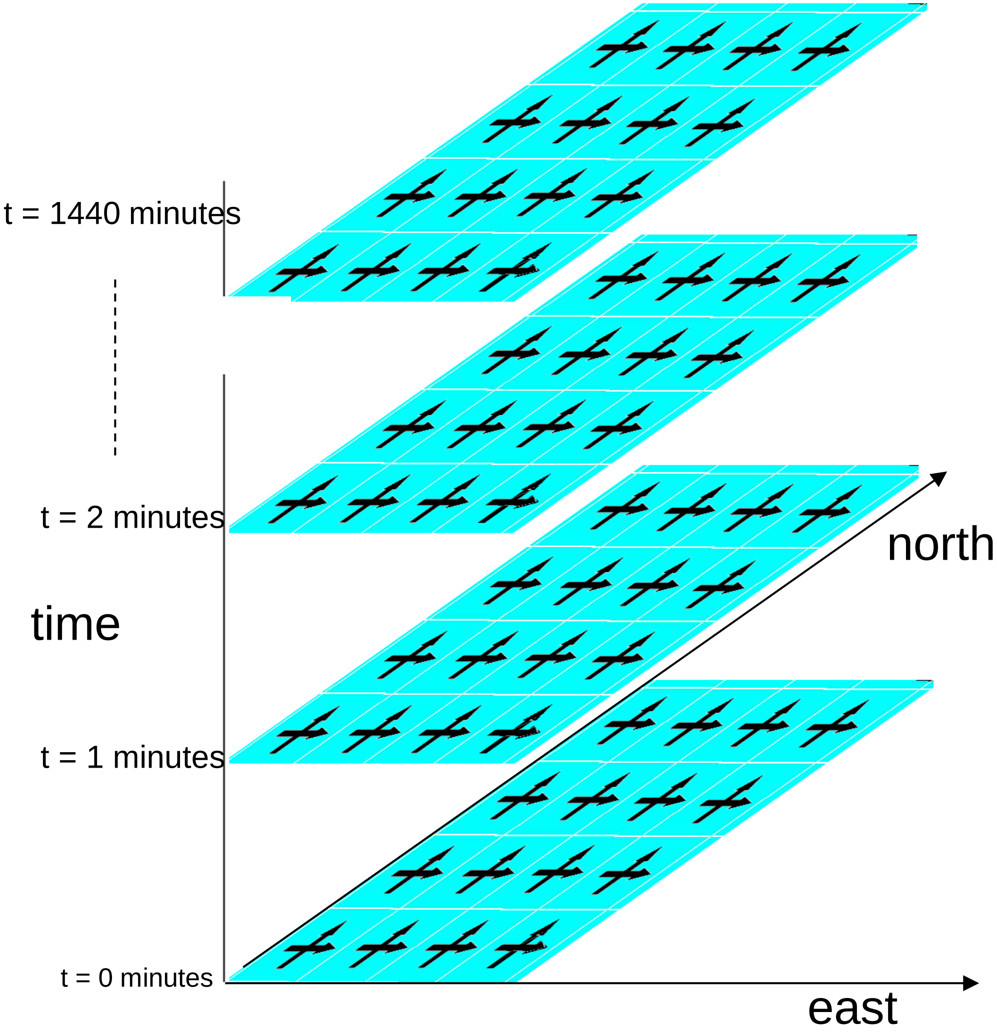
**from presentation of G. Freyer (2006)**

# New: **SIFT** – **S**hort-term **I**nundation **F**orecasting for **T**sunamis

•One 16 min x 16 min full Pacific ‘snapshot’ is stored for each minute of wave evolution starting at event time (t = 0) and ending 1 full day later. So, 1441 ‘snapshots’

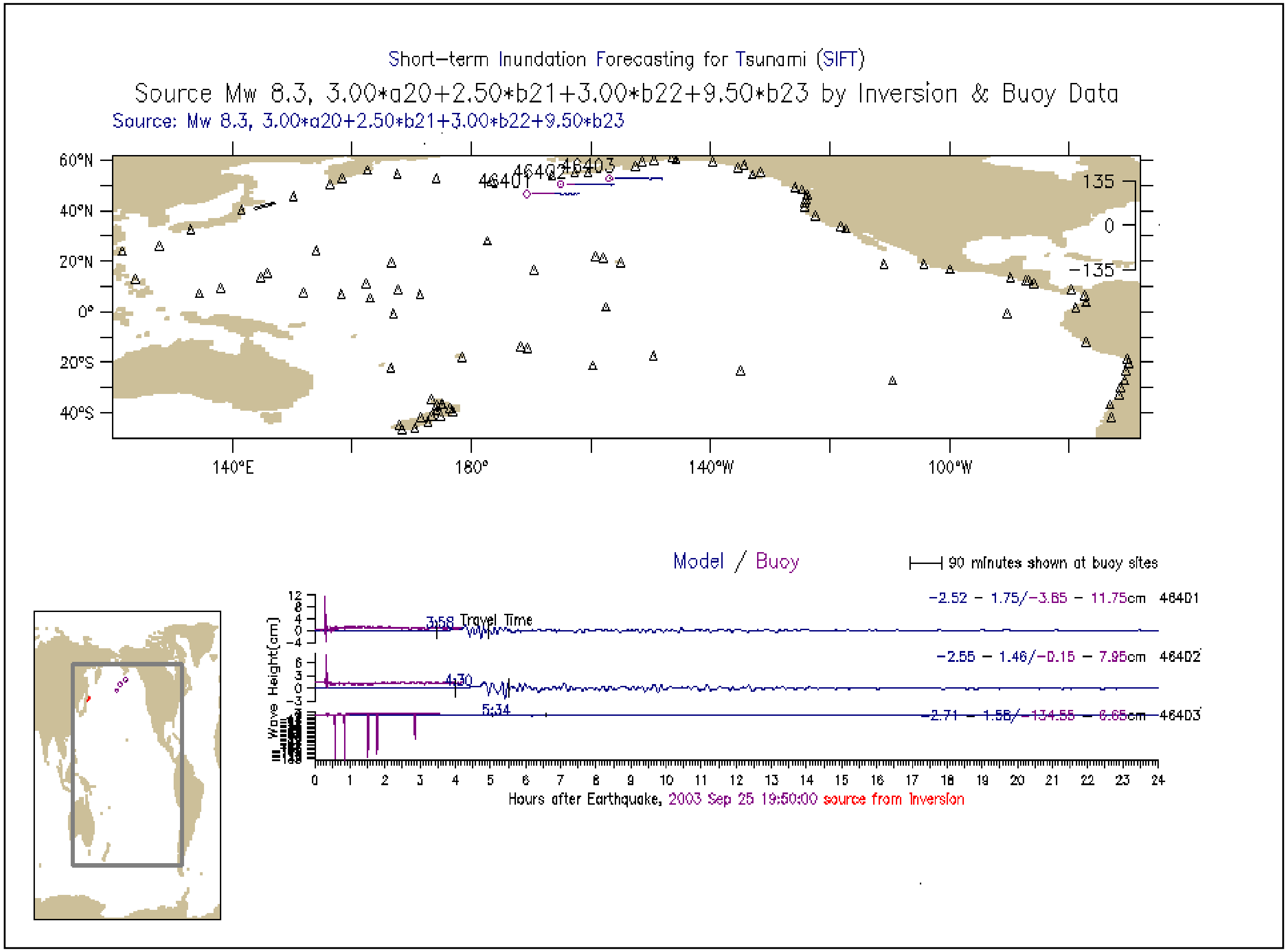
•3 variables per cell and roughly 200,000 rectangular cells cover the Pacific from 50S to 62N. This requires 0.6 million stored output values for each one-minute snapshot.

- Multiplying the above by 1441 one minute snapshots gives about 7 Gbytes of storage space required for each wave source.



**from presentation of G. Freyer (2006)**

# New: **SIFT** – **S**hort-term **I**nundation **F**orecasting for **T**sunamis

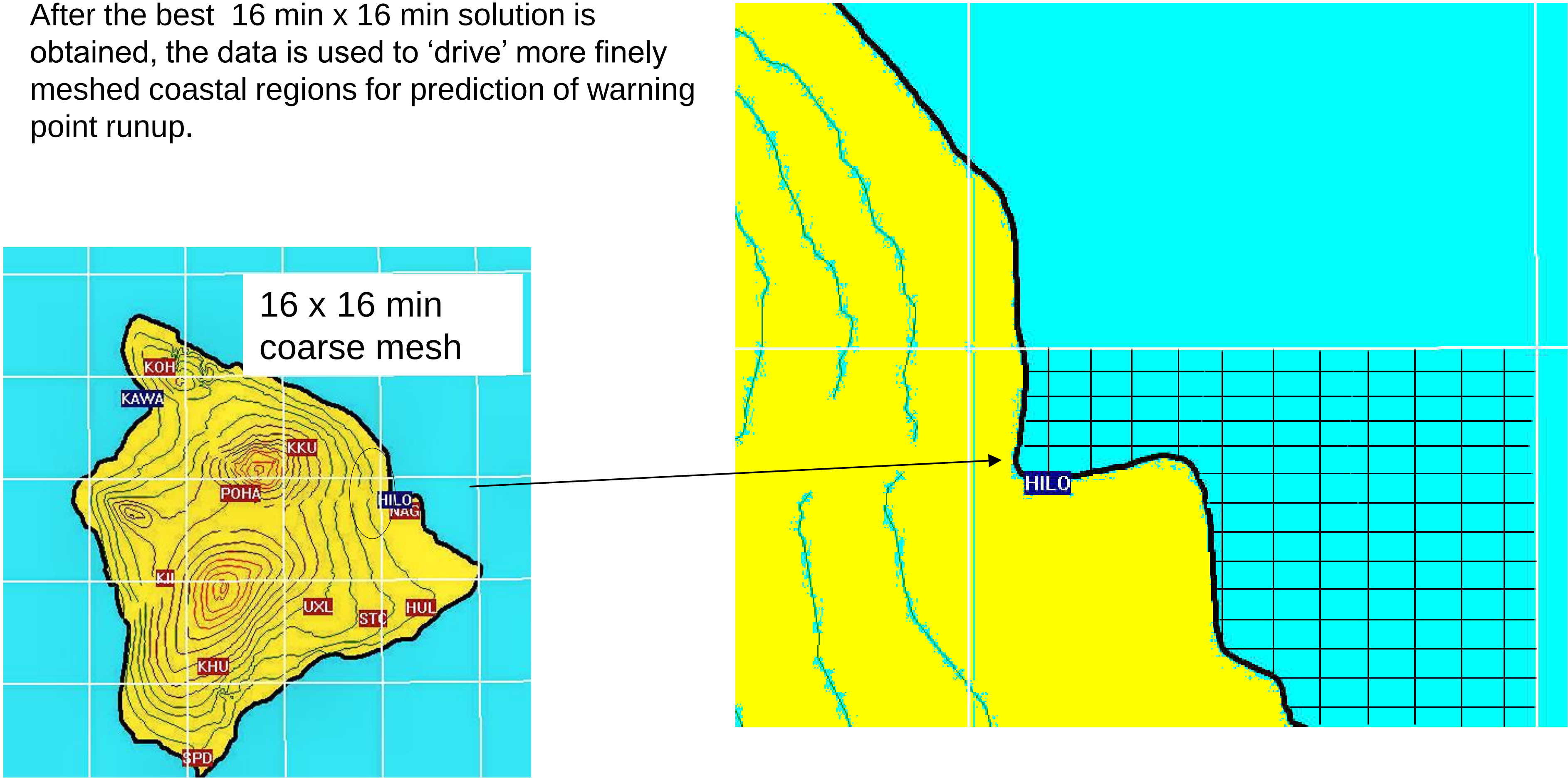


**from presentation of G. Freyer (2006)**

# Far-field: PTWC

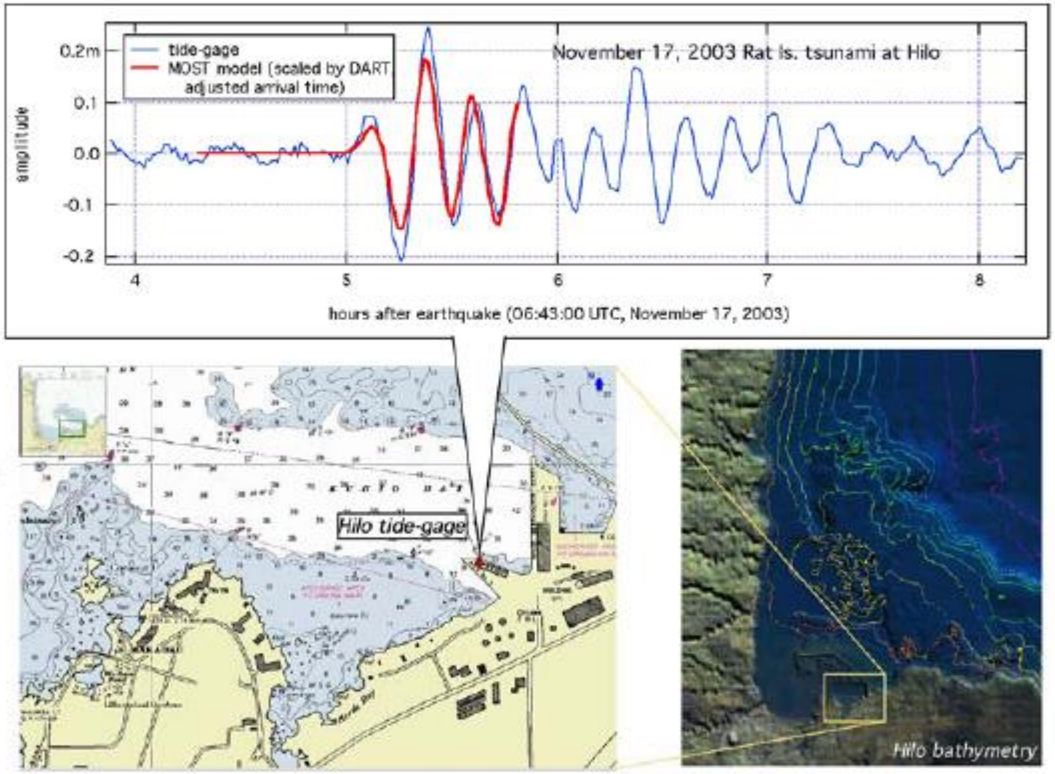
## New: **SIFT** – **S**hort-term **I**nundation **F**orecasting for **T**sunamis

After the best 16 min x 16 min solution is obtained, the data is used to ‘drive’ more finely meshed coastal regions for prediction of warning point runup.


16 x 16 min coarse mesh

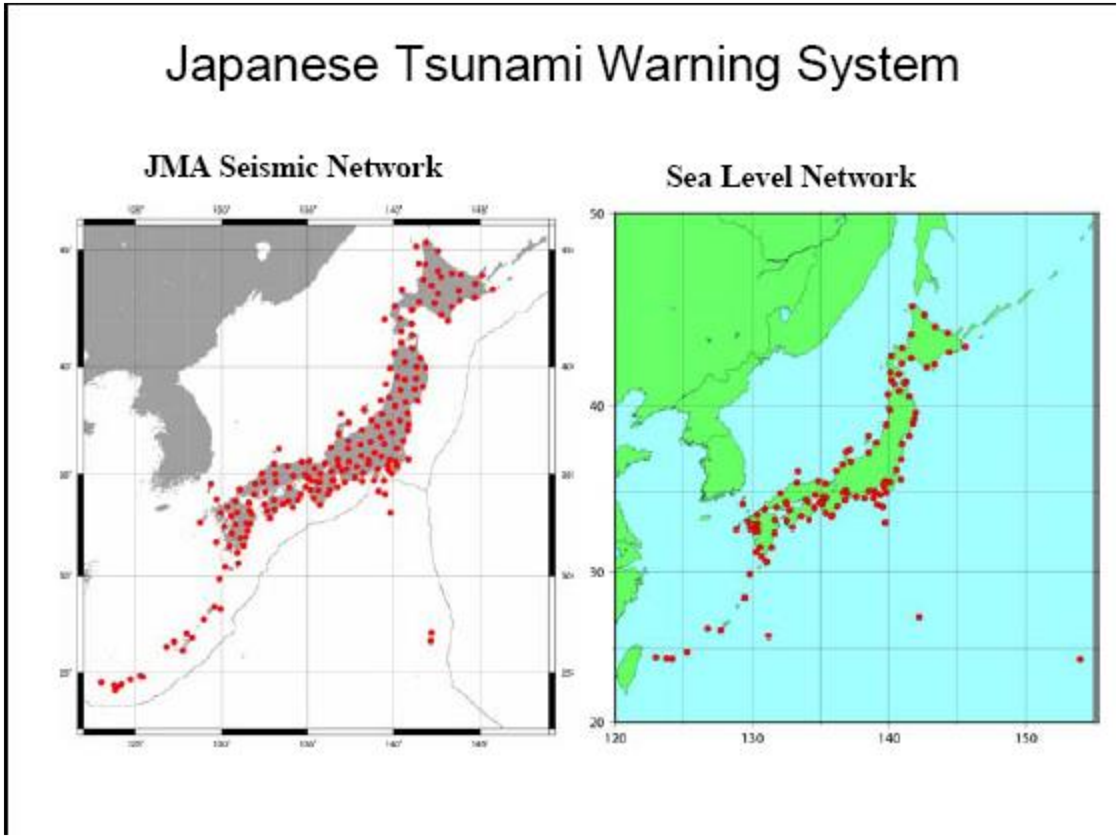
from presentation of G. Freyer (2006)

# New: **SIFT** – **S**hort-term **I**nundation **F**orecasting for **T**sunamis



**from presentation of McCreery (2005)**

# Near-field: Japan



## Japanese Tsunami Warning System

JMA Seismic Network

Sea Level Network

**Satake (2005)**

# Databank of virtual tsunamis

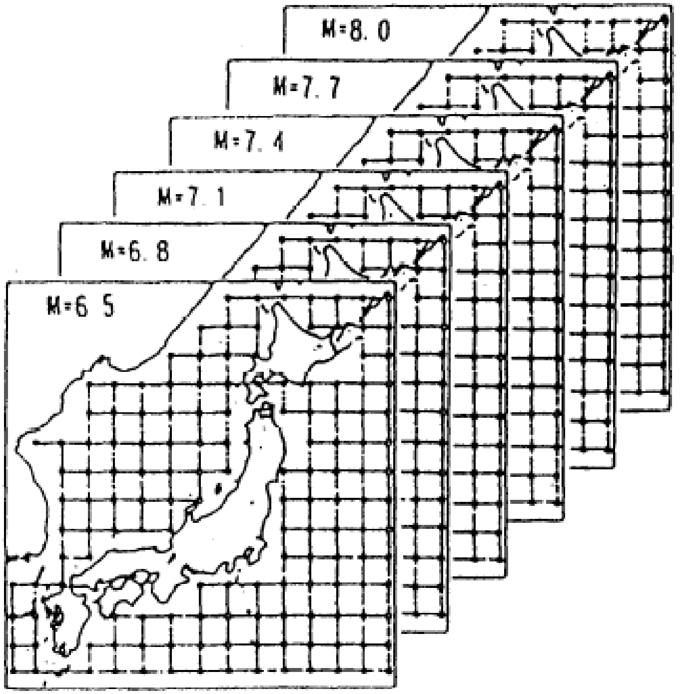
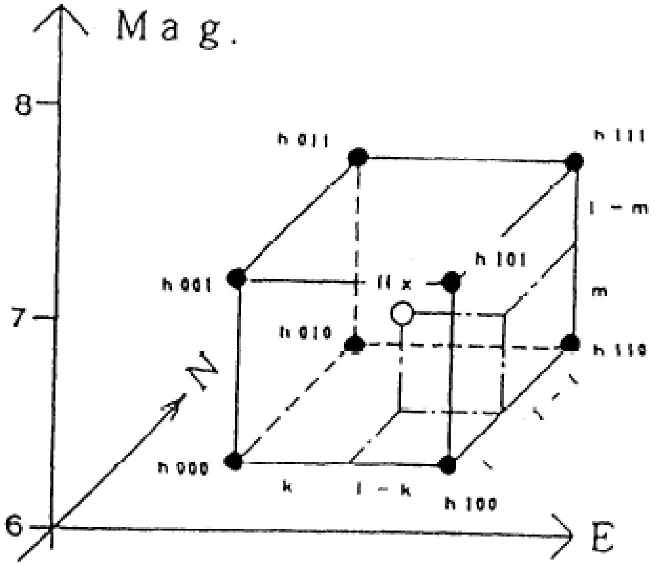


Figure 3 Archived Matrix of Pre-calculated Virtual Tsunamis



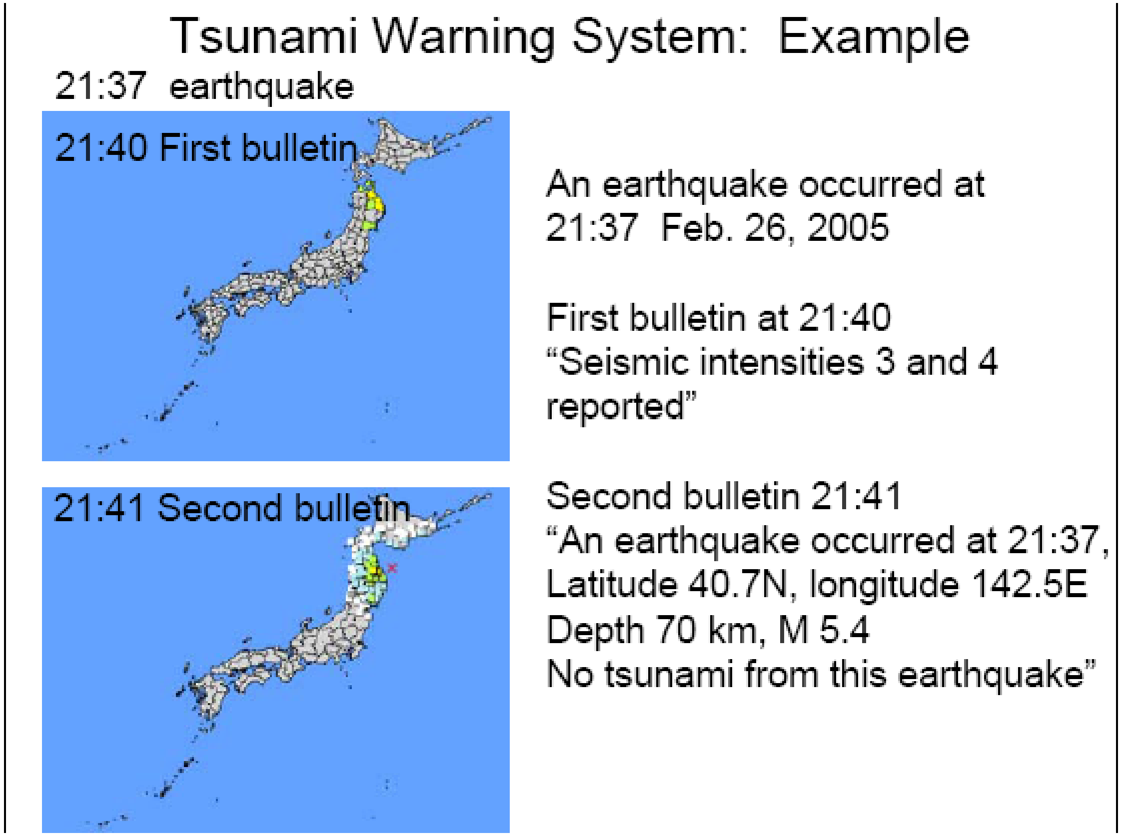
$$Hx = [(1-l)((1-k)h000 + kh100) + l((1-k)h010 + kh110)]^{1-m}$$
$$[(1-l)((1-k)h001 + kh100) + l((1-k)h010 + kh111)]^{m}$$

Figure 4 Interpolation Scheme among Virtual Tsunamis

**Science of Tsunami Hazards, Vol 17, No. 2 (1999) page 101**

**Furumoto et al. (1999)**

# Near-field: Japan



## Tsunami Warning System: Example

21:37 earthquake

21:40 First bulletin

An earthquake occurred at
21:37 Feb. 26, 2005

First bulletin at 21:40
“Seismic intensities 3 and 4 reported”

21:41 Second bulletin

Second bulletin 21:41
“An earthquake occurred at 21:37,
Latitude 40.7N, longitude 142.5E
Depth 70 km, M 5.4
No tsunami from this earthquake”

**Satake (2005)**

## Numerical Modeling for:

1. Tsunami early warning
2. Tsunami hazard assessment
3. Integrative testing of the TEWS
4. Personnel teaching and training

# Modeling hazard assessment

- Similar to earthquakes, tsunamis will take place in the future worldwide and collect their toll
- To minimize negative impact, society needs preparedness and early warning
- Preparedness starts from hazard assessment

# Tsunami hazard assessment: 2 approaches

**Deterministic scenario-based**

- Worst case scenarios
- Does not provide recurrence periods of tsunamis
- Challenges high-resolution modeling

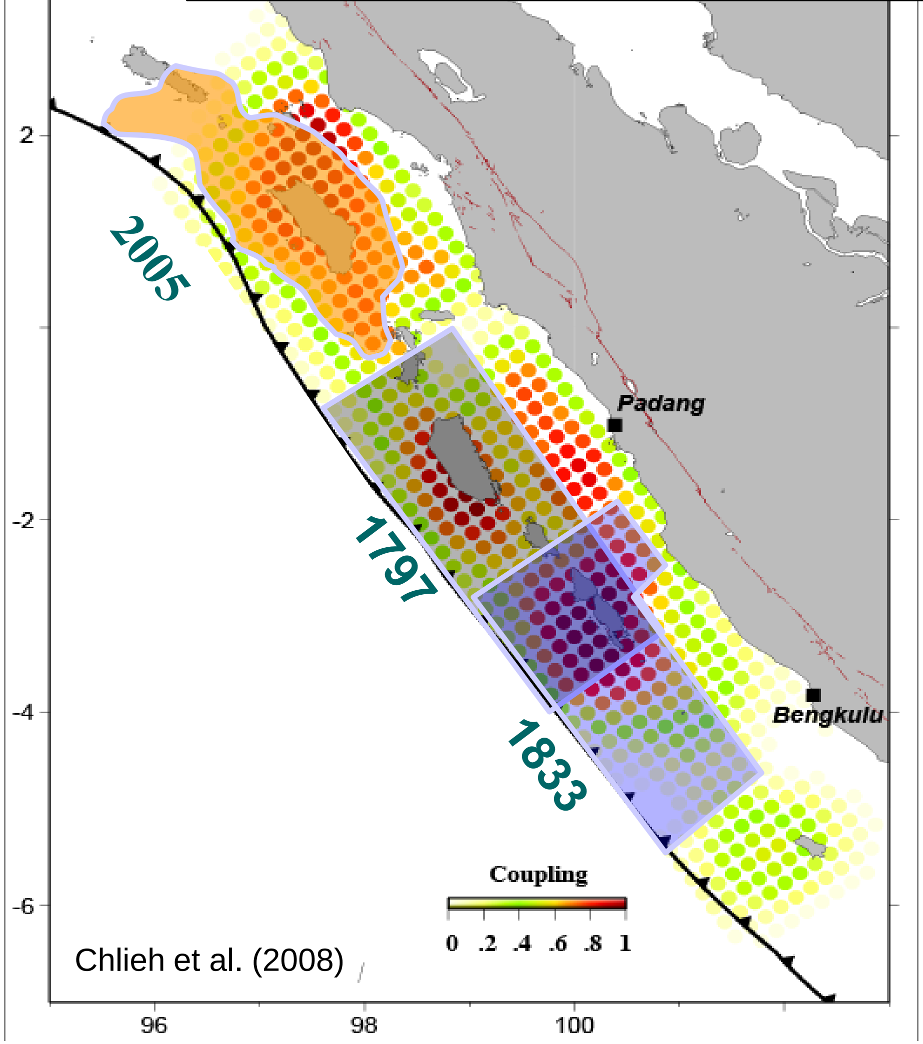
Oriented at coastal planning and early warning

**Probabilistic approach**

- Huge number of scenarios aggregated
- Provides recurrence periods of tsunamis of different wave heights (hazard curves)

Oriented at engineering and risk assessment

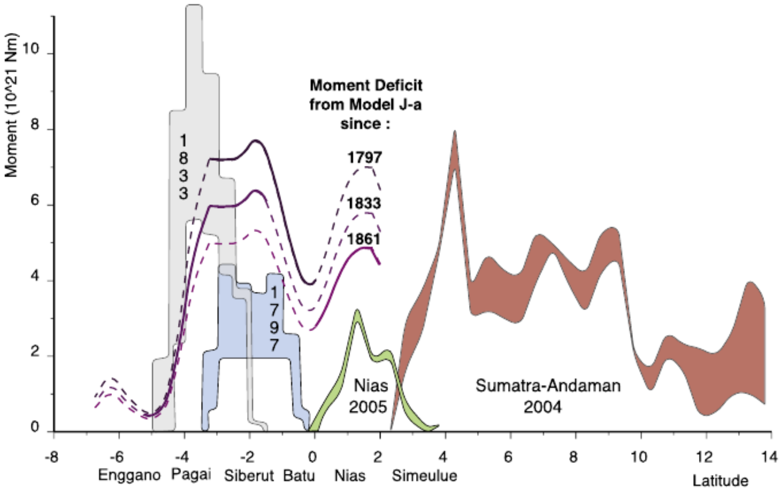
# Central Sumatra: Inter-plate coupling as revealed by geodetic and coral data



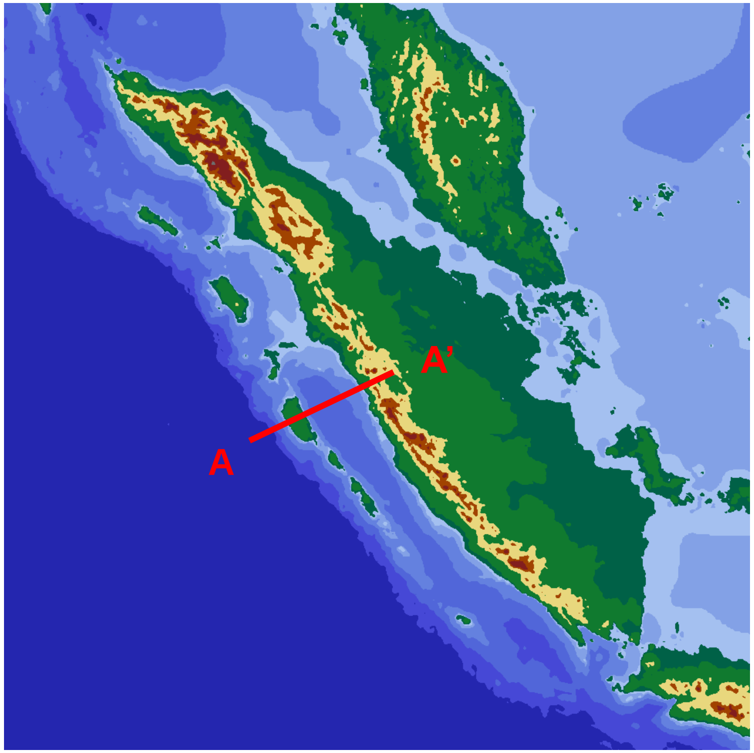
Chlieh et al. (2008)

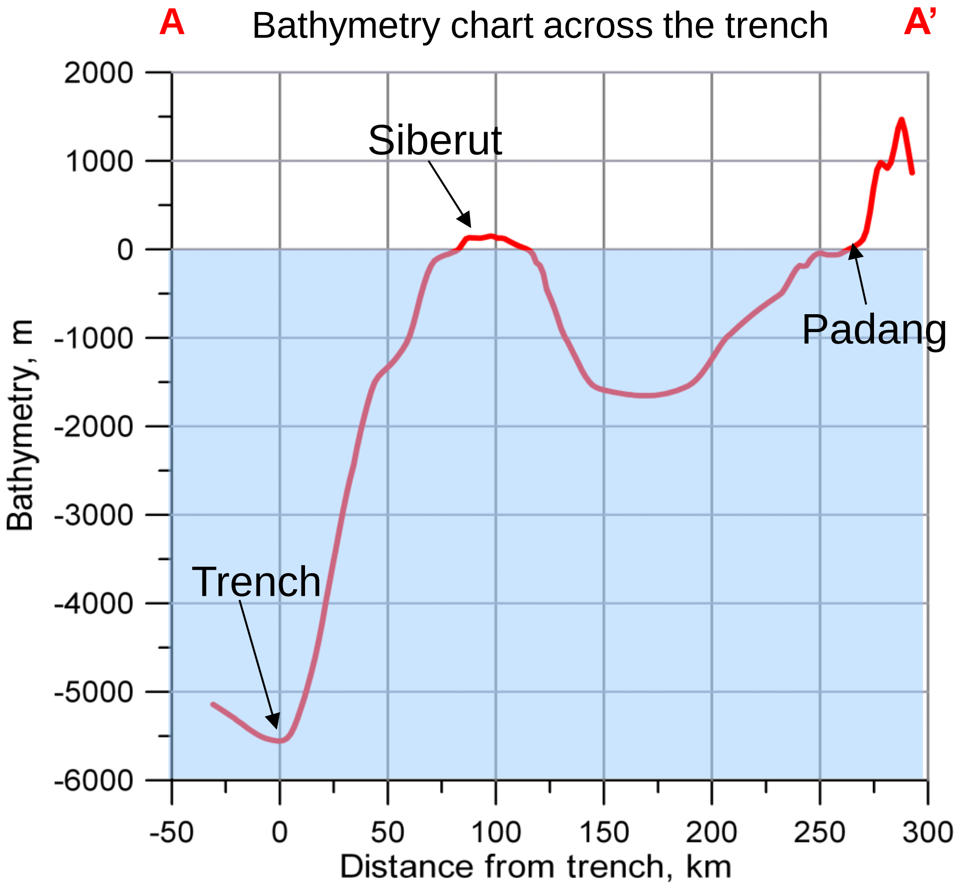
Full locking during ~200 years results in about:
200 y * 5.5 cm/y = 11 m
accumulated slip.

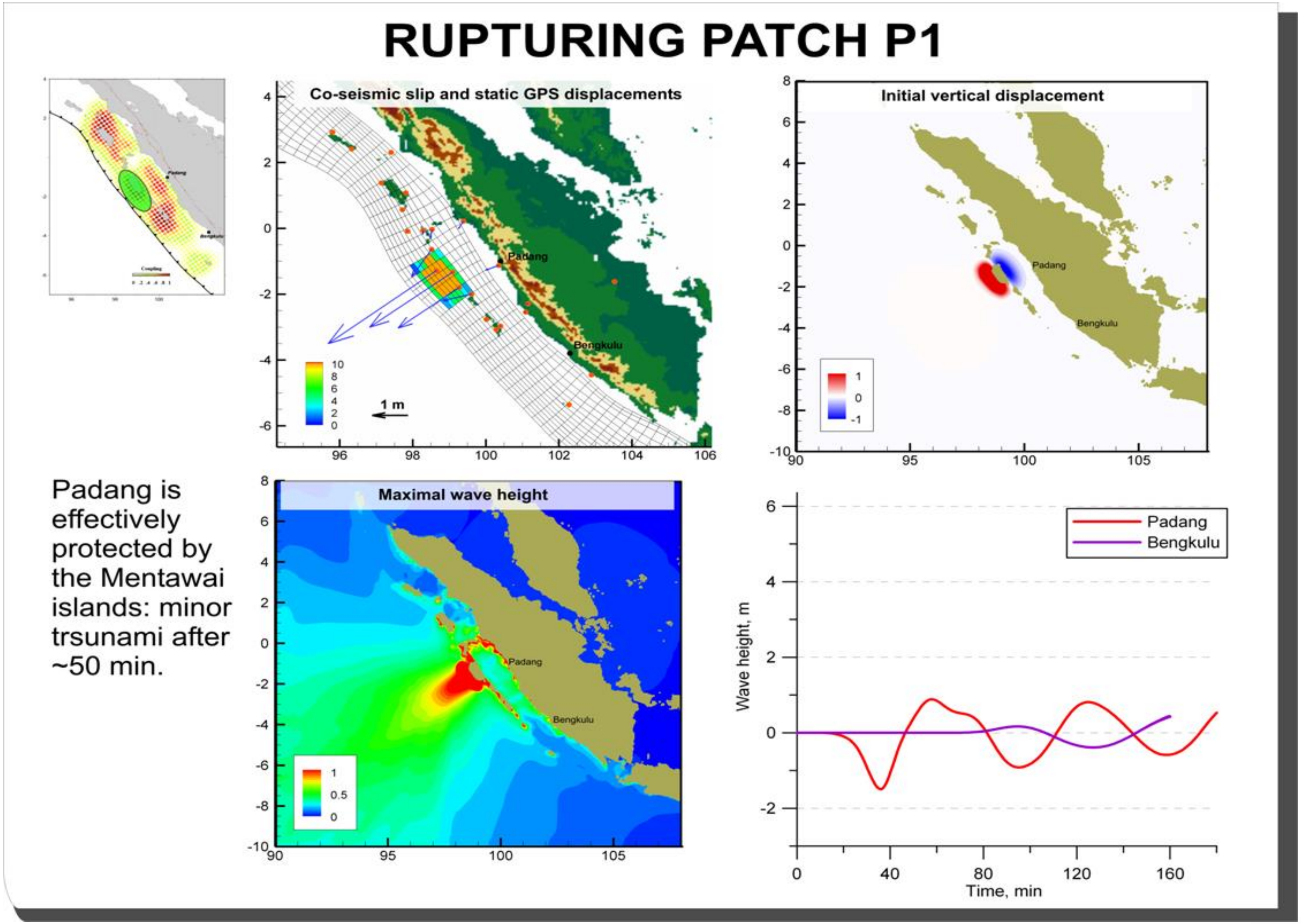
**Accumulated seismic moment deficit after Chlieh et al. (2008)**



# Bathymetry off Padang: An important player



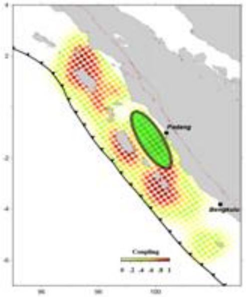


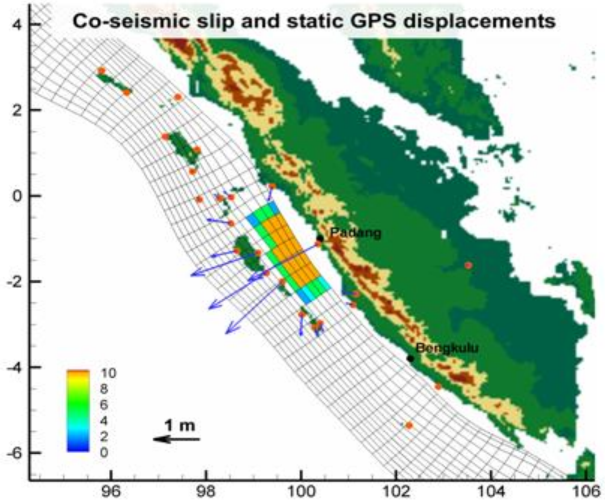


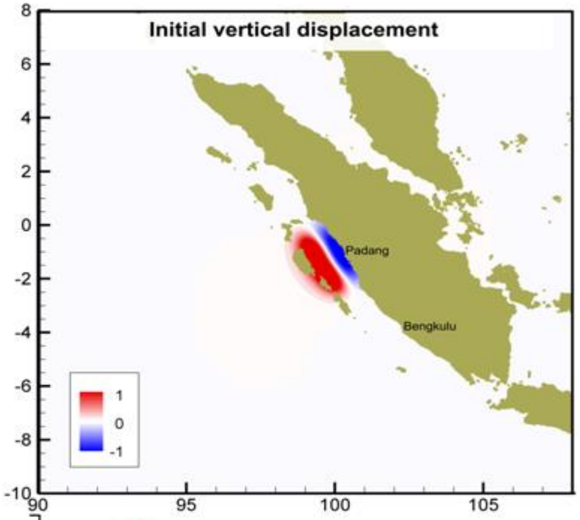
# RUPTURING PATCH P1

Padang is effectively protected by the Mentawai islands: minor trsunami after ~50 min.

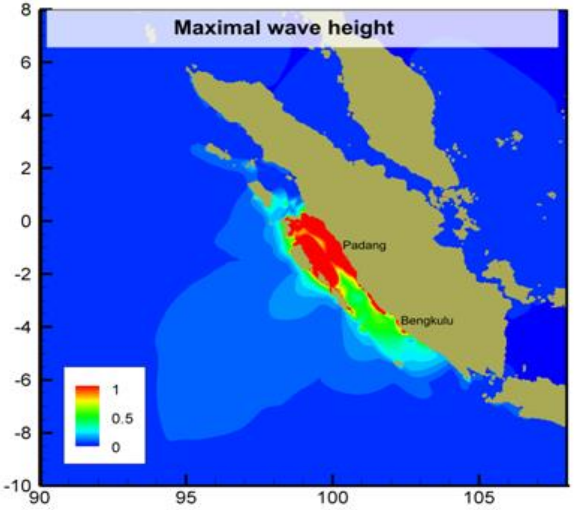
# RUPTURING PATCH P2

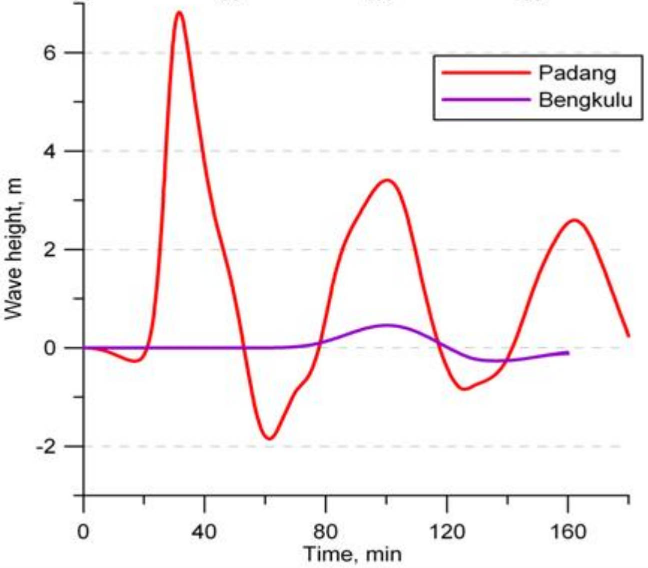




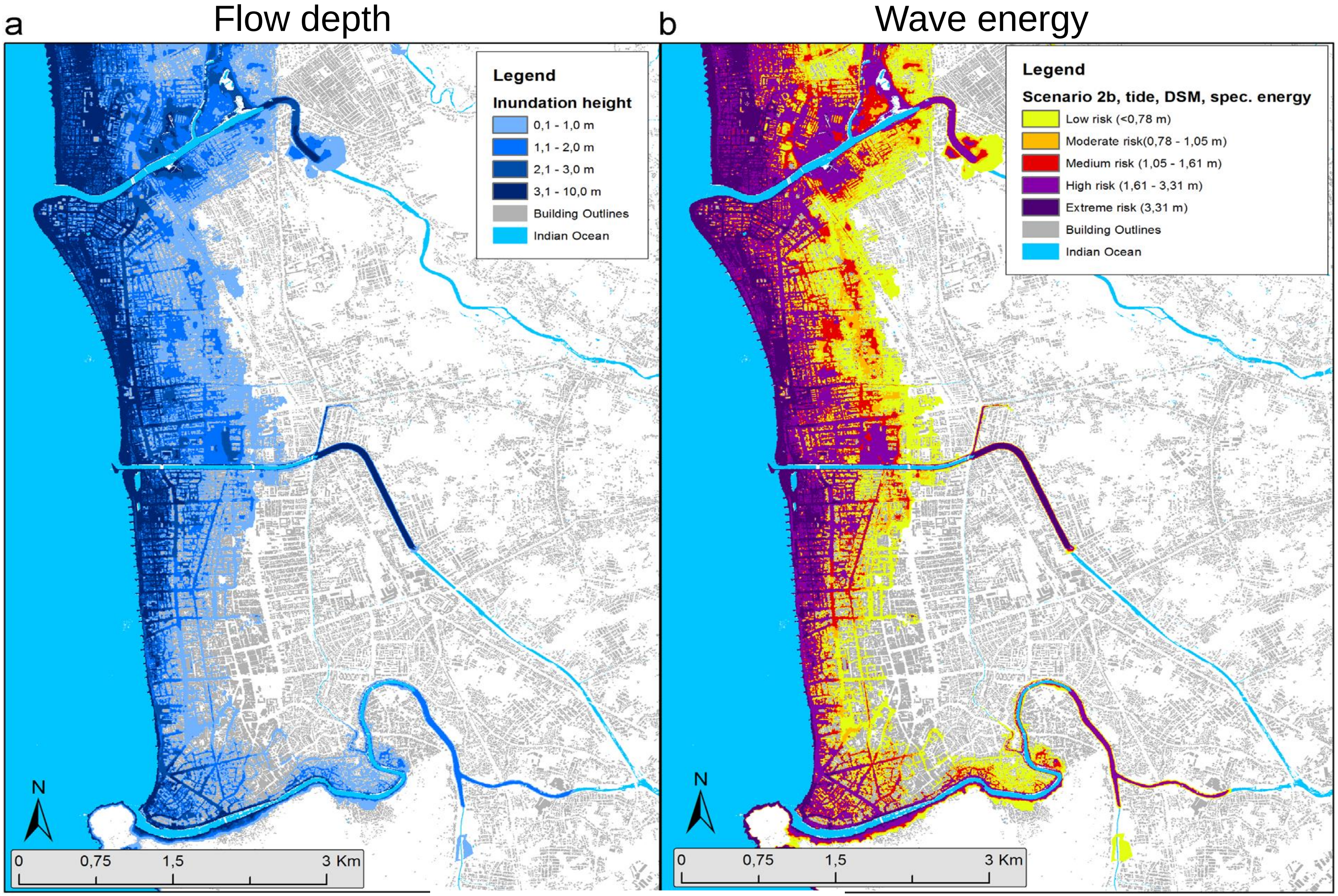


Tsunami energy is trapped between the Mentaway chain and Sumatran coast. Strong inundation after 30 min in Padang.





# High-resolution inundation modeling for city of Padang, Sumatra (Indonesia)



Taubenböck et al. 2013

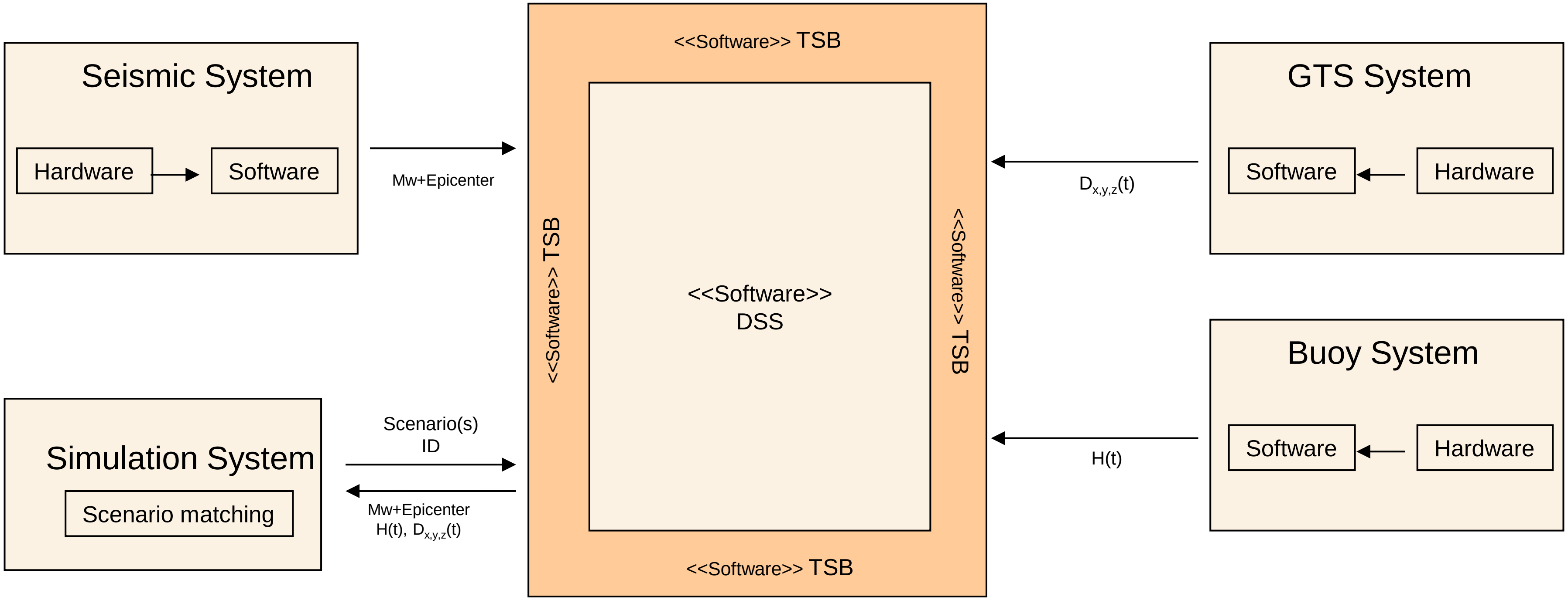
# Modeling for testing

## Numerical Modeling for:

1. Tsunami early warning
2. Tsunami hazard assessment
3. Integrative testing of the TEWS
4. Personnel teaching and training

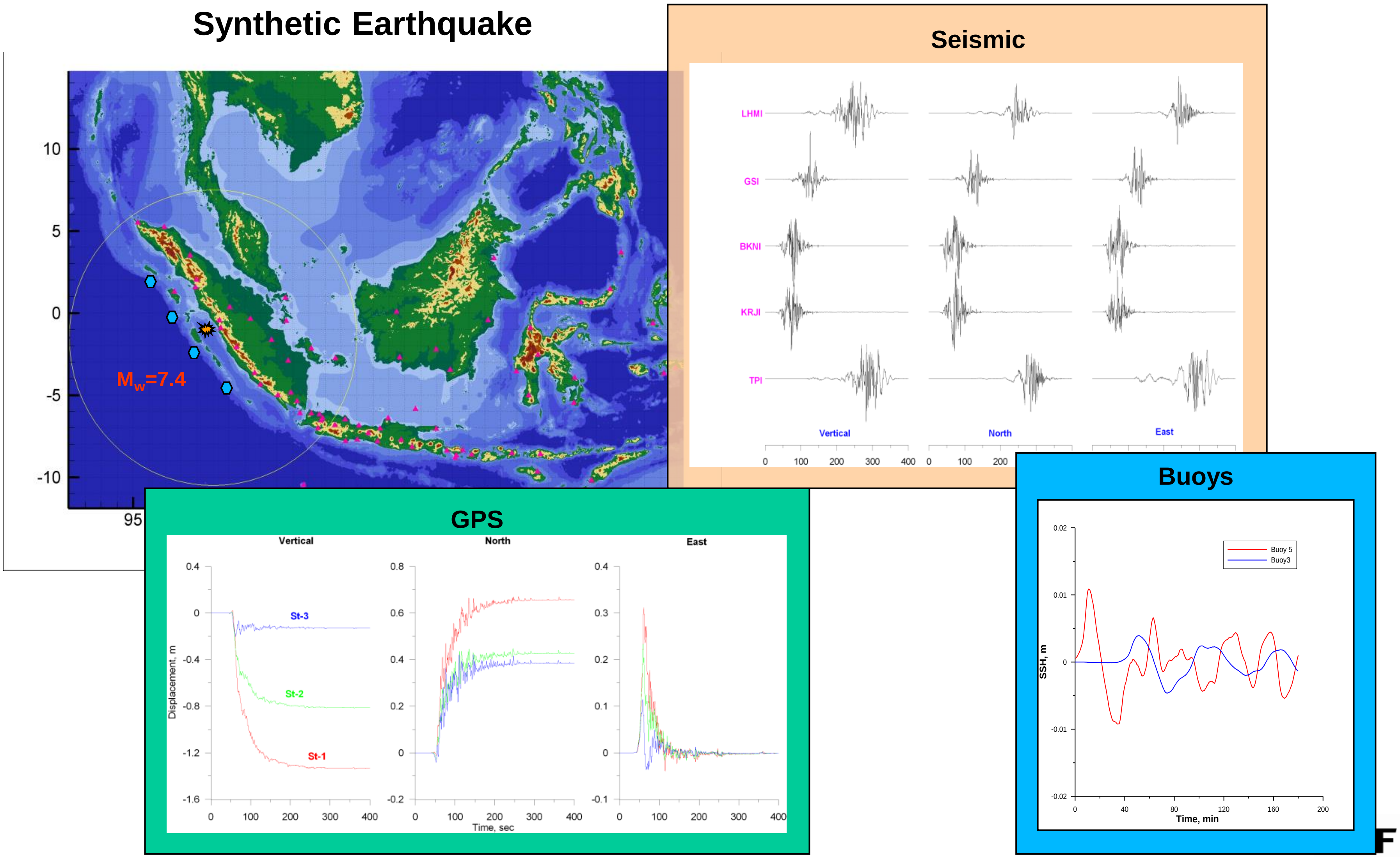
# Modeling for testing

## Simplified System Architecture



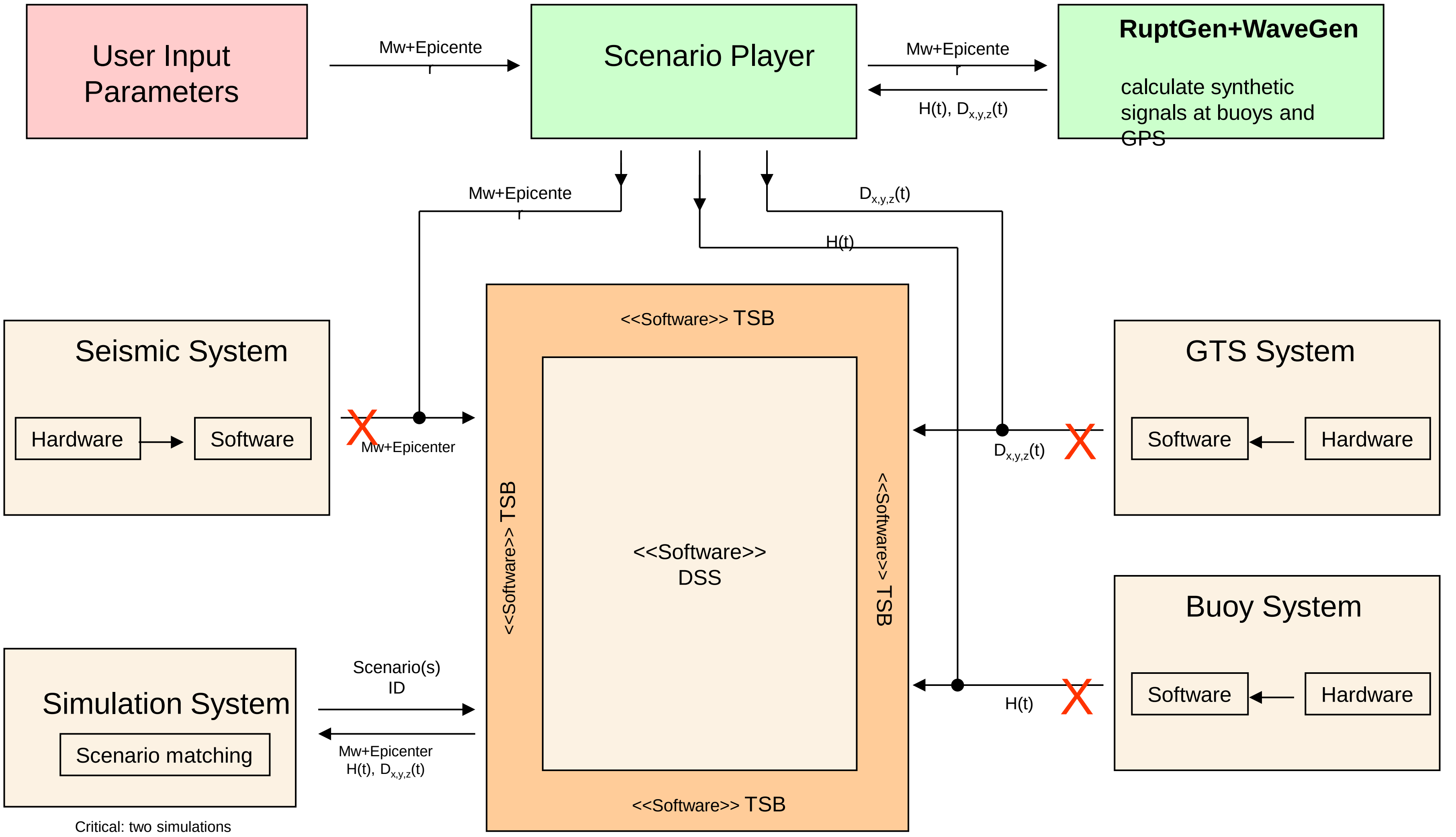
# Modeling for testing

## Complete synthetic scenarios of rupture and corresponding sensor responses



# Modeling for testing

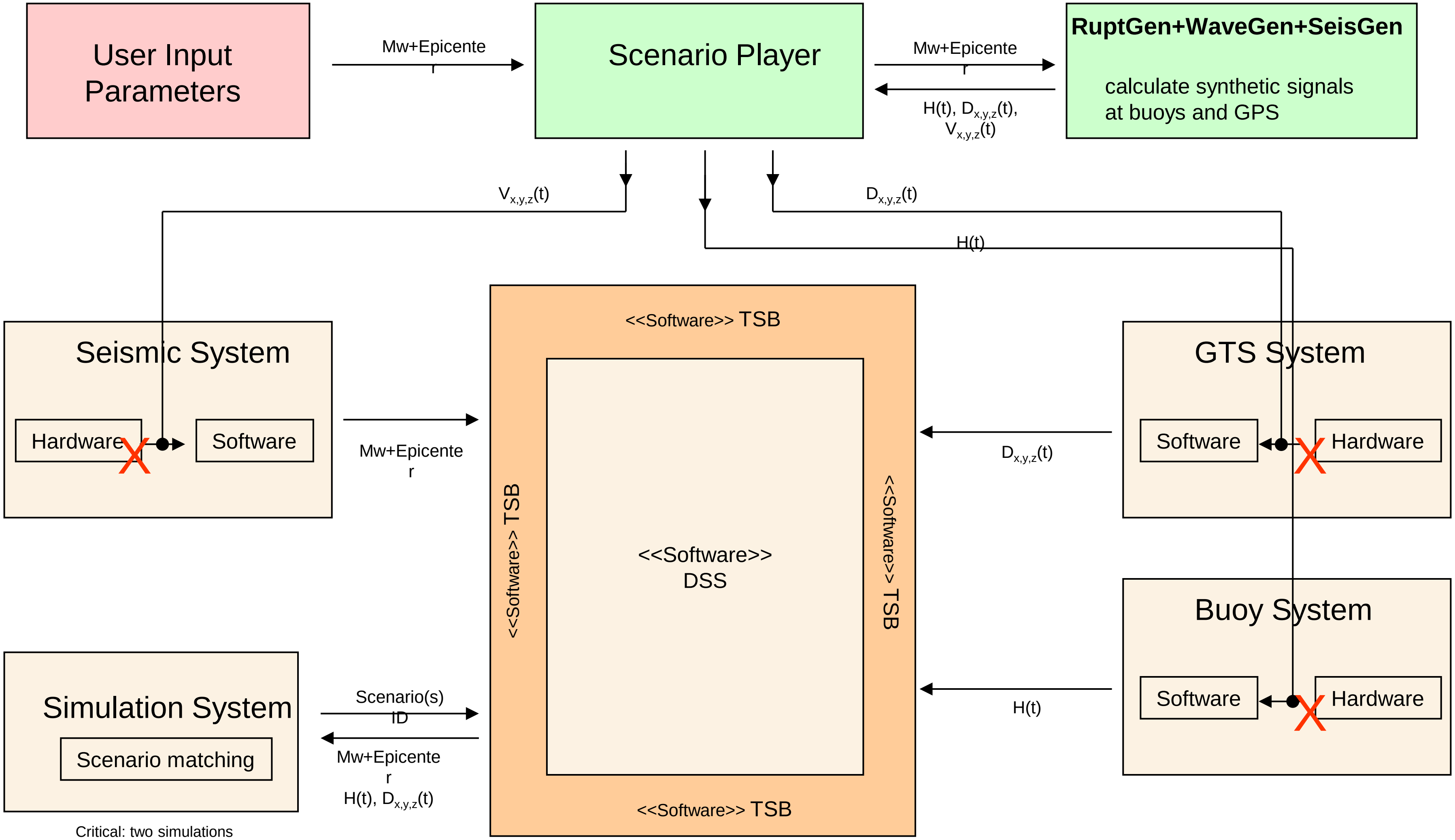
**Simulated sensor data are realistic and consistent with each other in time and magnitude.**
**Simulation is probably the only way to supply TSB and DSS with feeds of fully consistent sensor data of various nature.**



Critical: two simulations compete with each other.

# Modeling for testing

**Simulated sensor data are realistic and consistent with each other in time and magnitude.**
**Simulation is probably the only way to supply TSB and DSS with feeds of fully consistent sensor data of various nature.**



User Input Parameters

Mw+Epicenter

Scenario Player

Mw+Epicenter

**RuptGen+WaveGen+SeisGen**

calculate synthetic signals at buoys and GPS

H(t), $D_{x,y,z}(t)$, $V_{x,y,z}(t)$

$V_{x,y,z}(t)$

$D_{x,y,z}(t)$

H(t)

Seismic System

Hardware

Software

Mw+Epicenter

<<Software>> TSB

<<Software>> DSS

GTS System

Software

Hardware

$D_{x,y,z}(t)$

Buoy System

Software

Hardware

H(t)

Simulation System

Scenario matching

Scenario(s) ID

Mw+Epicenter

H(t), $D_{x,y,z}(t)$

Critical: two simulations compete with each other.

# Modeling for teaching and training

## Numerical Modeling for:

1. Tsunami early warning
2. Tsunami hazard assessment
3. Integrative testing of the TEWS
4. Personnel teaching and training

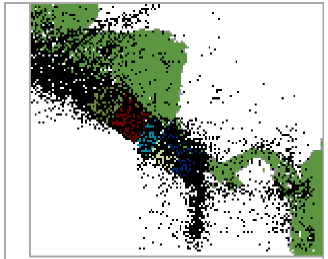
# Probabilistic approach: seismic PTHA

a. Based on extensive local catalogs of tsunami: **empirical**

b. Based on earthquake catalogs and tsunami simulations: **computational**

# Steps of probabilistic analysis

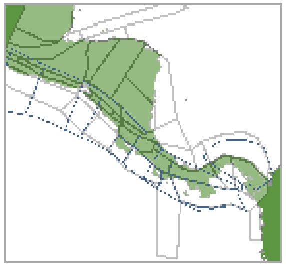
Seismic catalog



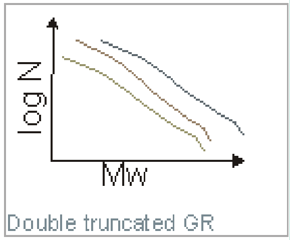
+ geophysical datasets

→

Seismo-tectonic zones



Seismic activity within zones

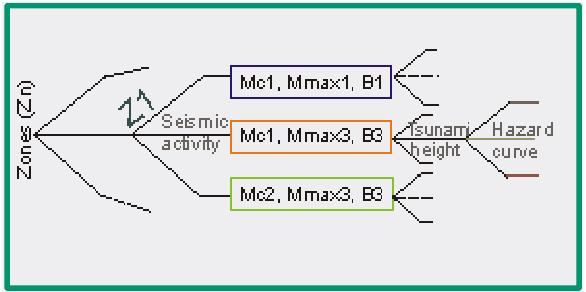


Double truncated GR

→

100000 years catalog

Synthetic catalog



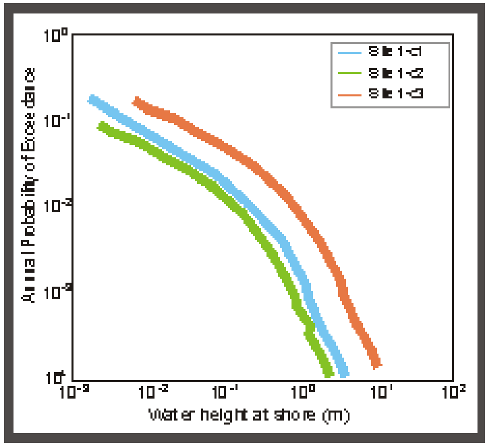
Logic trees: epistemic uncertainty

→

Tsunami propagation: Magnitude threshold for simulations: Mw>6

Wave heights at shore: Tsunami Green Function

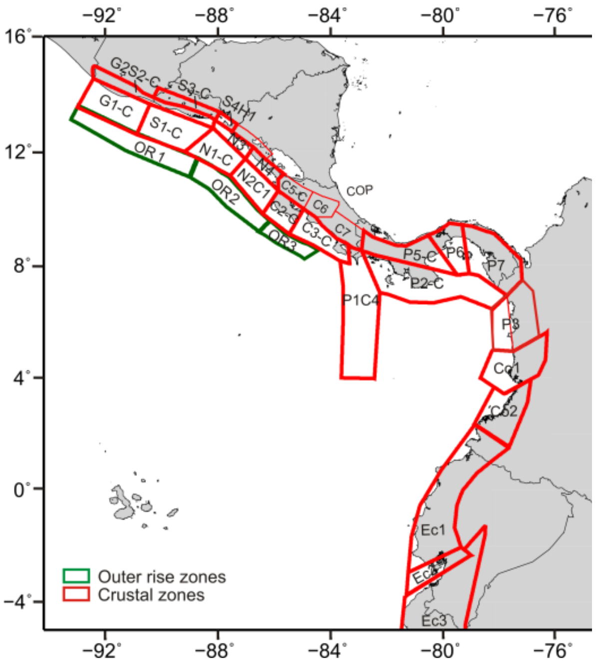
→



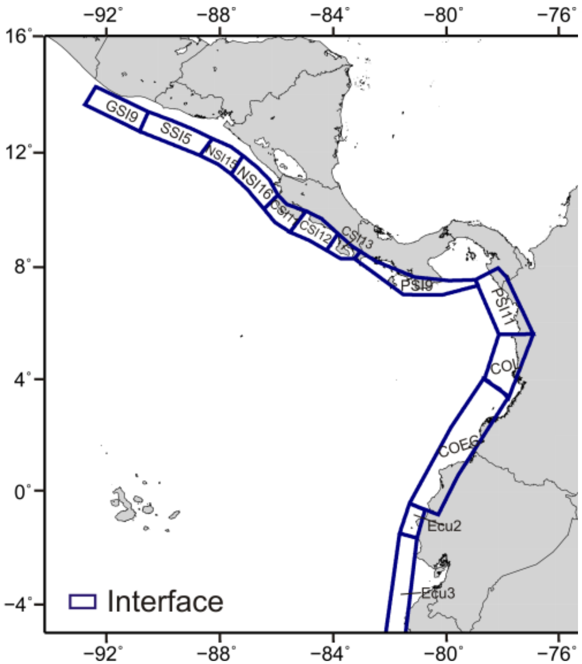
Hazard curves only Zone 1

# Seismic source zones in Central America

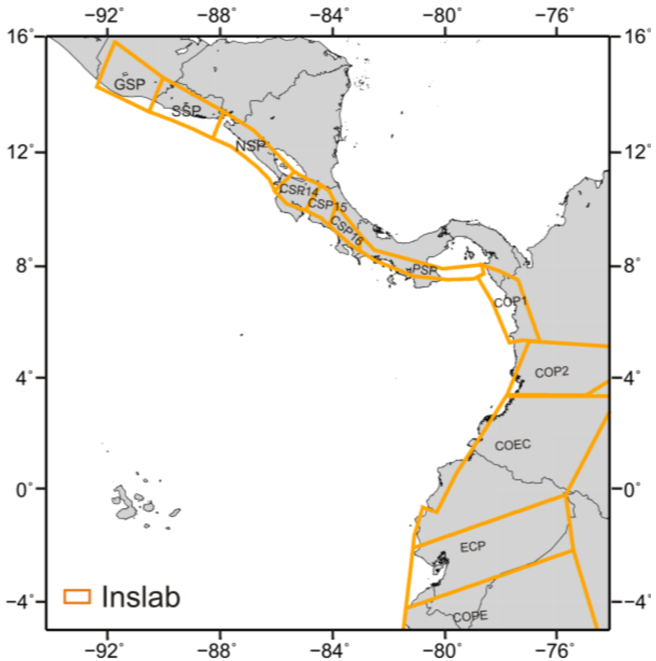
1. Outer rise and crustal upper plate



2. Subduction plate interface



3. Intra-slab



Based on seismic catalog: CAT2011

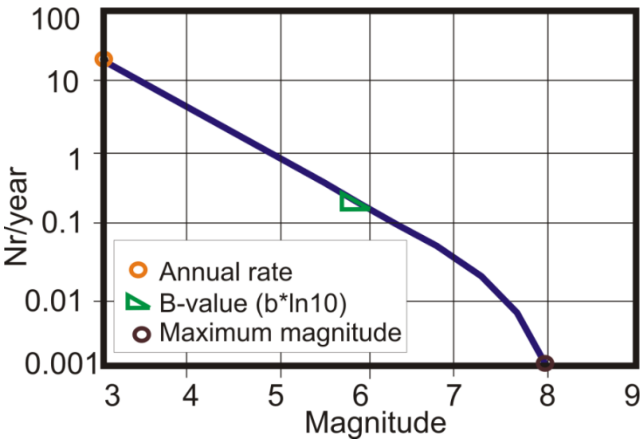
# Deriving parameters for Monte-Carlo simulations

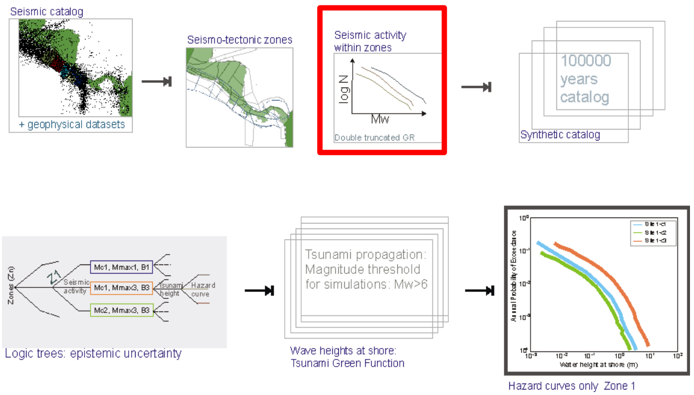
Gutenberg-Richter type magnitude–frequency relation:
log N= a-bM

$$\mathbf{P}[\mathbf{E} > \mathbf{Mi}] = \frac{e^{-\beta(Mi)} - e^{-\beta(Mmax)}}{e^{-\beta(Mmin)} - e^{-\beta(Mmax)}}$$

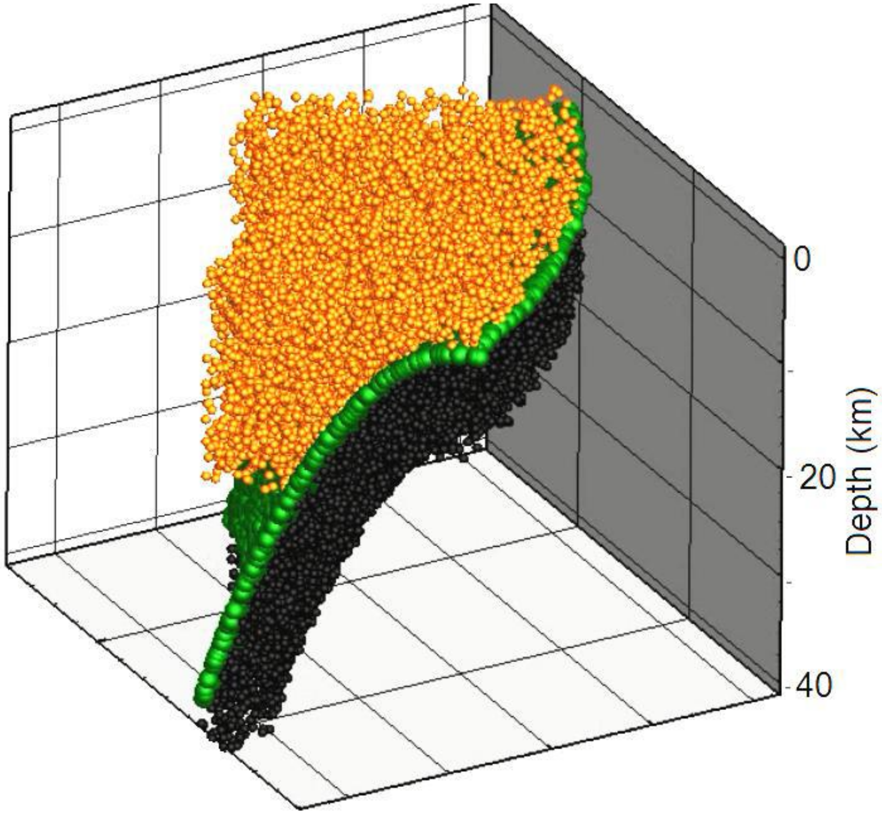
| | |
|---|---|
| CRN1<br>b: 0.77+/-0.03<br>a: 5.6<br>Mc: 3.7+/-0.05 | NIC1<br>b: 1.3 +/-0.03<br>a: 6.73<br>Mc: 4.6+/-0.01 |
| CRC2<br>b: 1.01 +/-0.03<br>a: 6.73<br>Mc: 3.6 +/-0.02 | CRC3<br>b: 1.01 +/-0.03<br>a: 5.82<br>Mc: 3.7+/-0.06 |

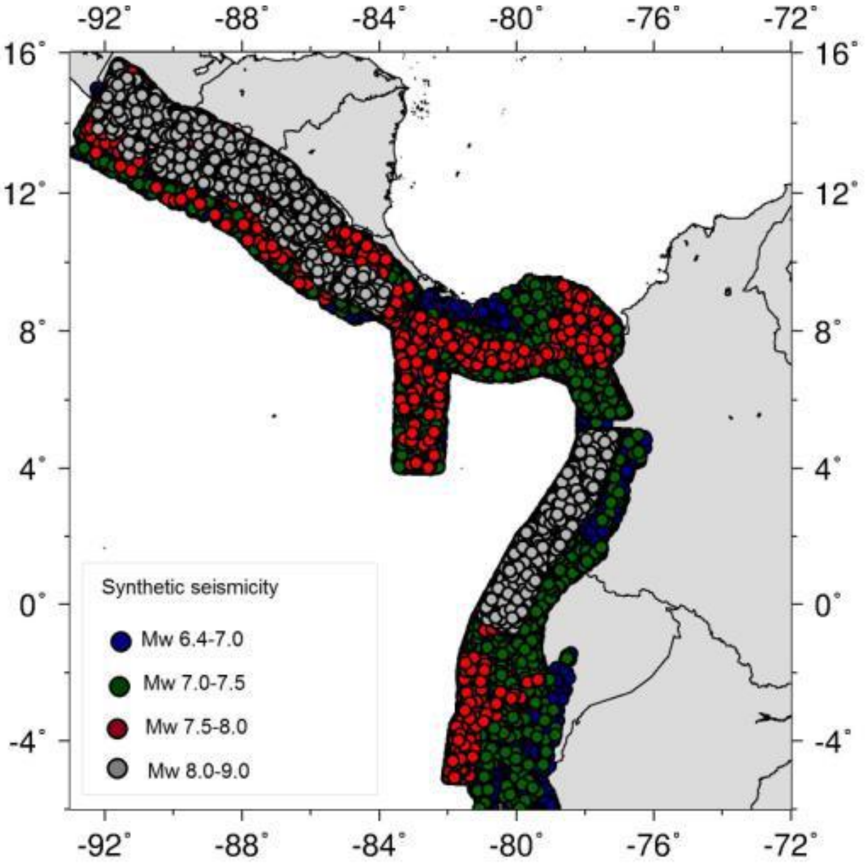
**Mc**: magnitude of completeness of seismic catalogue
**β**=b*ln10

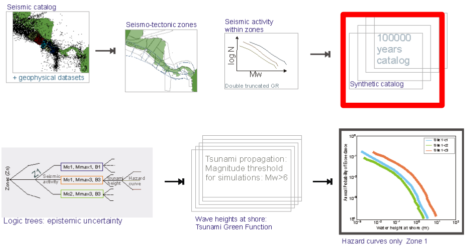




# Synthetic catalog for 100 000 years







Seismic catalog

+ geophysical datasets

Seismo-tectonic zones

Seismic activity within zones

Double truncated GR

100000 years catalog

Synthetic catalog

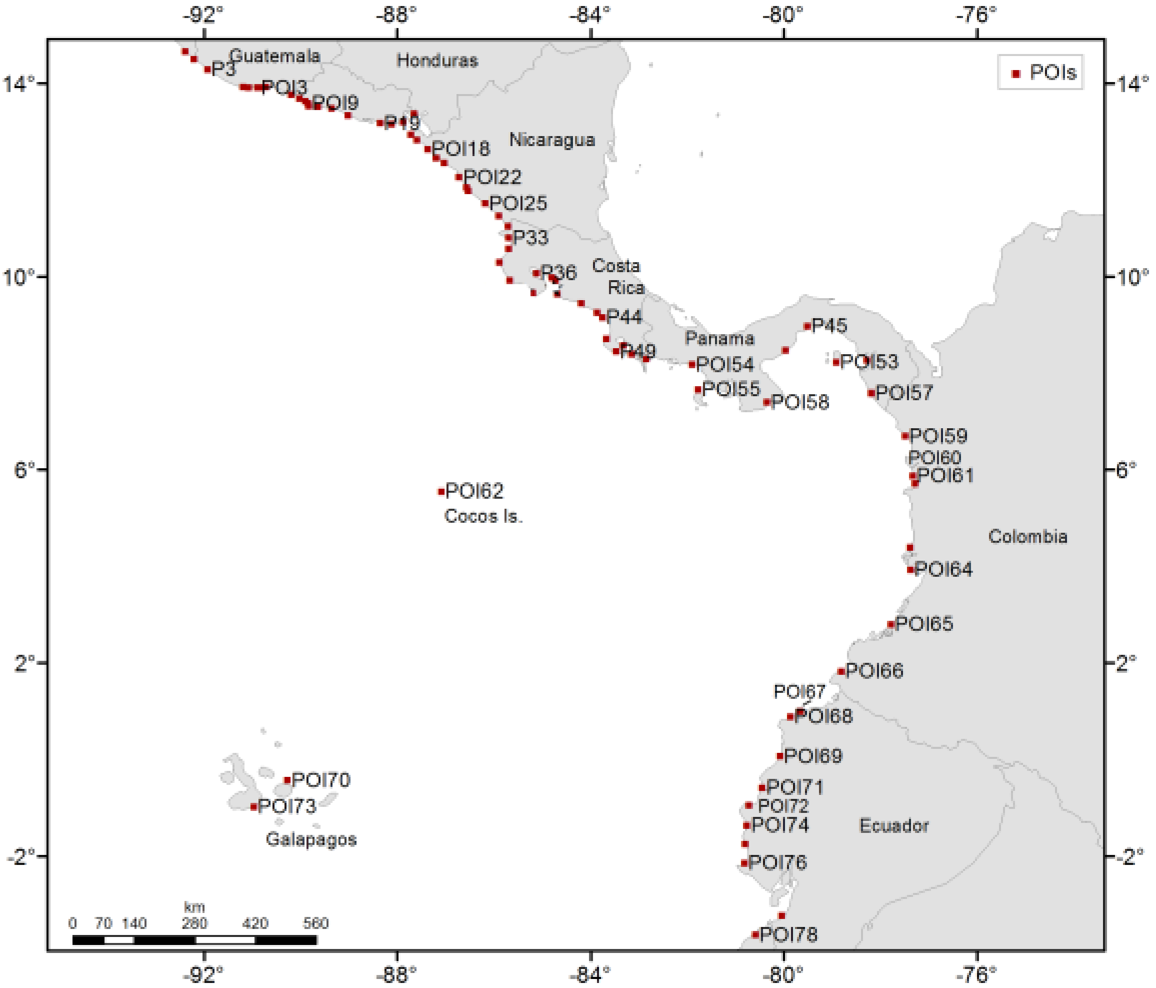
Logic trees: epistemic uncertainty

Tsunami propagation: Magnitude threshold for simulations: Mw>6

Wave heights at shore: Tsunami Green Function

Hazard curves only Zone 1

# Selecting Points Of Interest (POI)

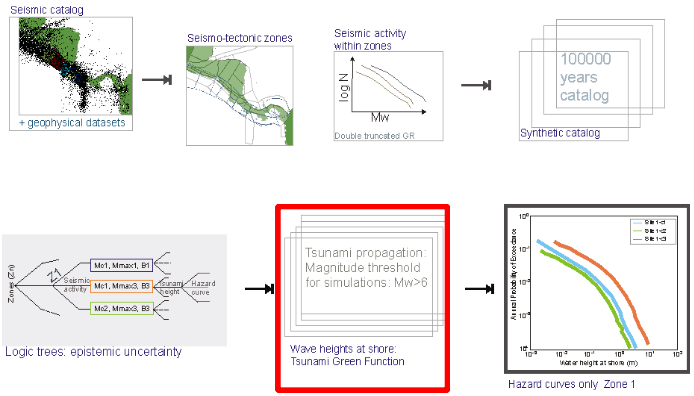


# Computing wave heights at POIs for all the sources

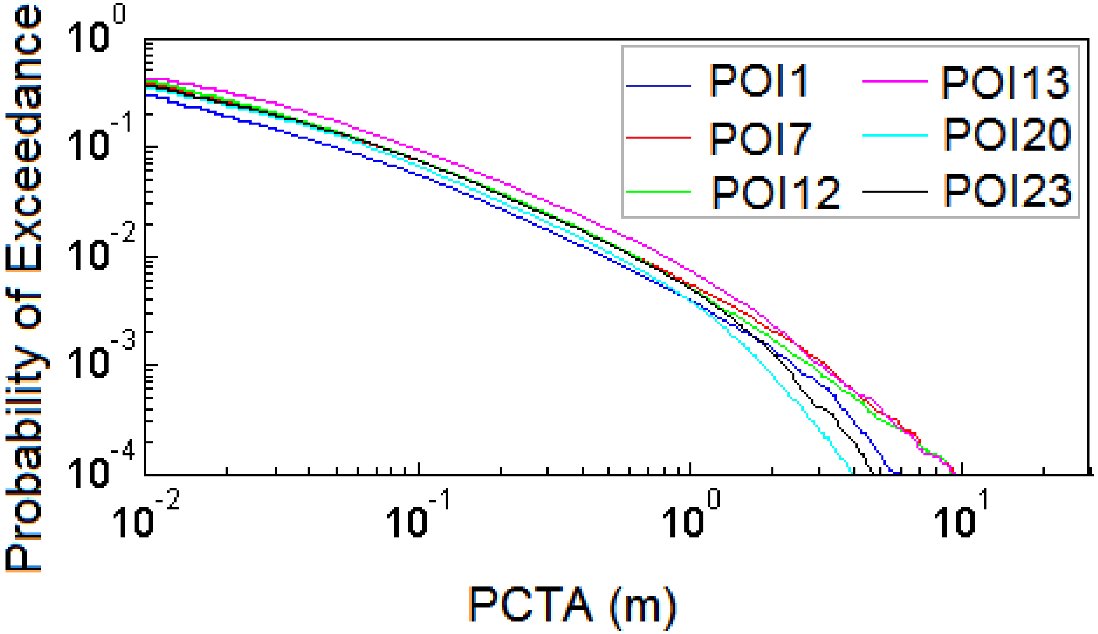
Extensive computational task: Hunderts of thousands of scenarios

Different approaches:

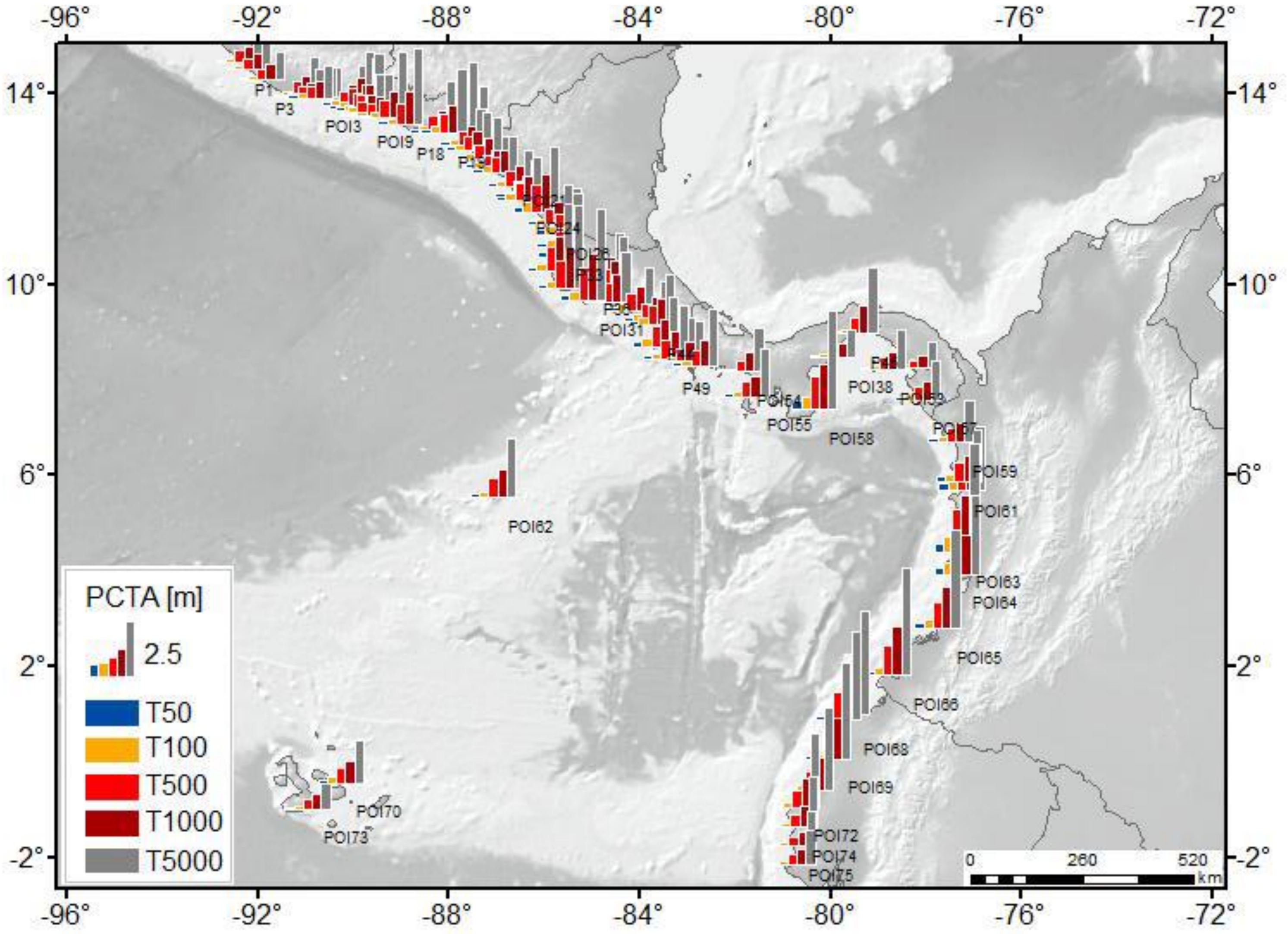
- Using pre-computed linear Green‘s Functions
- Direct computations of offshore amplitudes with fast (linear) numerical algorithms



# Hazard results: hazard curves @ selected POIs



# Wave heights expected @ different return periods



# Wave heights expected @ different return periods: Another view