Modeling of Fluid Flow in Porous Media

Marius J. Walter

GFZ Potsdam

March 8, 2016

Why is porous flow important for geodynamic problems?

- Affecting melt temperatures
- Slab dehydration and intermediate seismic activity
- Mechanical weakening along tectonic interfaces
- ...

Why is porous flow important for geodynamic problems?

- Affecting melt temperatures
- Slab dehydration and intermediate seismic activity
- Mechanical weakening along tectonic interfaces
- ...

Why is porous flow important for geodynamic problems?

- Affecting melt temperatures
- Slab dehydration and intermediate seismic activity
- Mechanical weakening along tectonic interfaces
- ...

Why is porous flow important for geodynamic problems?

- Affecting melt temperatures
- Slab dehydration and intermediate seismic activity
- Mechanical weakening along tectonic interfaces
- ...

My research question

What controls the migration of the magmatic arc at subduction zones?

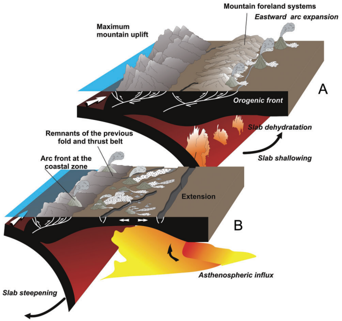


Figure: Folguera et al., 2011

Learn more about that on Friday ...

My research question

avi

2 phase flow

Assumption I

Melt is distributed in a connected network of pores between a solid matrix.

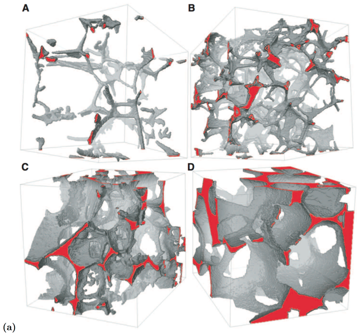
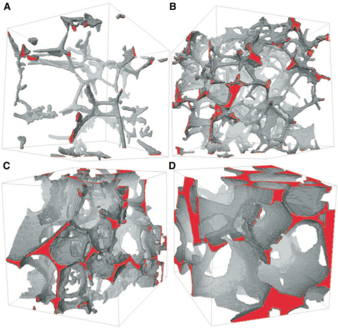


Figure: Zhu et al., 2011

2 phase flow

Assumption II

The viscosity of the solid matrix and fluid are orders of magnitude different. For example: $\eta_s = 1e20 Pa \cdot s$ and $\eta_f = 10 Pa \cdot s$



(a)

Figure: Zhu et al., 2011

2 phase flow

Assumption III

Solid and fluid phases are considered to be intrinsically incompressible materials. All compressibility in the model is accounted for by changes in melt fraction, due to compression or dilation.

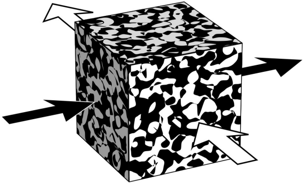


Figure: Katz et al., 2014

Derivaton

Stokes Flow

$$\eta_s \left(\nabla \boldsymbol{v_s} + (\nabla \boldsymbol{v_s})^T \right) - \nabla \boldsymbol{P} = \overline{\rho} g$$

$$\nabla \cdot \boldsymbol{v_s} = -\frac{\boldsymbol{P}}{\xi^*_{eff}}$$

$+$

Darcy Flow

$$q = -\phi \triangle v = -K_D \left(\nabla P_f + \rho_f g \right)$$

$+$

a little bit of math...

2 phase flow: equations

$$\nabla \cdot \bar{\tau} - \nabla \boldsymbol{P_f} - \nabla \boldsymbol{P_c} = \overline{\rho} g$$

$$\nabla \cdot \boldsymbol{v_s} - \nabla \cdot K_D \nabla \boldsymbol{P_f} = \nabla \cdot K_D \rho_f g$$

$$\nabla \cdot \boldsymbol{v_s} = -\frac{\boldsymbol{P_c}}{\xi^*_{eff}}$$

$$\frac{D\phi}{Dt} = (1 - \phi)\nabla \cdot \boldsymbol{v_s}$$

SLIM 3D + 2 phases: equations

2 phase flow

$$\nabla \cdot \bar{\tau} - \nabla \boldsymbol{P_f} - \nabla \boldsymbol{P_c} = \overline{\rho} g$$

$$\nabla \cdot \boldsymbol{v_s} - \nabla \cdot K_D \nabla \boldsymbol{P_f} = \nabla \cdot K_D \rho_f g$$

$$\nabla \cdot \boldsymbol{v_s} = -\frac{\boldsymbol{P_c}}{\xi^*_{eff}}$$

1 phase flow

Porosity = 0%

$$\nabla \cdot \bar{\tau} - \nabla \boldsymbol{P_f} = \overline{\rho} g$$

$$\nabla \cdot \boldsymbol{v_s} = 0$$

Porosity = 100%

$$-\nabla \cdot K_D \nabla \boldsymbol{P_f} = \nabla \cdot K_D \rho_f g$$

Melt-dependence of rheology: non-linear!

Permeability

$K_\phi = K_0 \phi^3$

K_ϕ ... Permeability, K_0 ... Reference Permeability, ϕ ... Porosity

Solid Viscosity

$\eta = \eta_0 exp(-\alpha_\phi \phi)$

η ... Solid Viscosity, η_0 ... Background Viscosity, α_ϕ ... fluid weaking factor

Bulk Viscosity

$\epsilon = \eta_0 \phi^{-n}$ ϵ ... Bulk Viscosity

Melt-dependence of rheology: non-linear!

Permeability

$K_\phi = K_0 \phi^3$

K_ϕ ... Permeability, K_0 ... Reference Permeability, ϕ ... Porosity

Solid Viscosity

$\eta = \eta_0 exp(-\alpha_\phi \phi)$

η ... Solid Viscosity, η_0 ... Background Viscosity, α_ϕ ... fluid weaking factor

Bulk Viscosity

$\epsilon = \eta_0 \phi^{-n}$ ϵ ... Bulk Viscosity

Melt-dependence of rheology: non-linear!

Permeability

$K_\phi = K_0 \phi^3$

K_ϕ ... Permeability, K_0 ... Reference Permeability, ϕ ... Porosity

Solid Viscosity

$\eta = \eta_0 exp(-\alpha_\phi \phi)$

η ... Solid Viscosity, η_0 ... Background Viscosity, α_ϕ ... fluid weaking factor

Bulk Viscosity

$\epsilon = \eta_0 \phi^{-n}$ ϵ ... Bulk Viscosity

Solitary Wave Benchmark

What's a Benchmark?

Comparison of the numerical results obtained solving the system of linear equations with ...

- analytical solutions
- results of physical (analogue) experiments
- numerical results from other(well-established) codes
- general physical considerations

Solitary Wave Benchmark

Why should you benchmark your numerical model?

Solitary Wave Benchmark

Idea

- Two-phase flow produces melt fraction instabilities: solitary waves.
- Solitary waves propagate with constant speed and without changing shape.

Implementation

- A initial solitary wave (for porosity) is imported into a (pseudo) 1D profile and propagates upward.
- Numerical errors are the deviation from the analytical derived speed and the initial shape of the solitary wave

Solitary Wave Benchmark

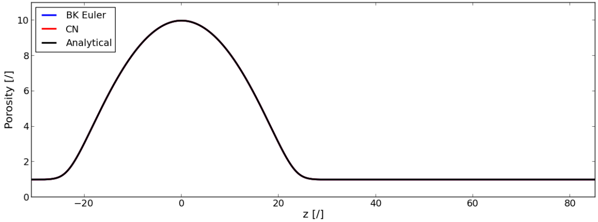
Idea

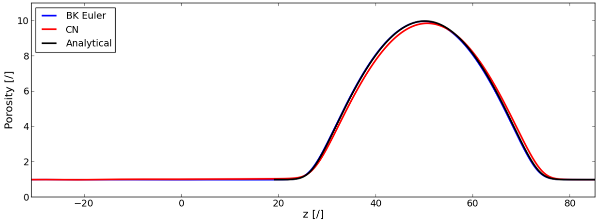
- Two-phase flow produces melt fraction instabilities: solitary waves.
- Solitary waves propagate with constant speed and without changing shape.

Implementation

- A initial solitary wave (for porosity) is imported into a (pseudo) 1D profile and propagates upward.
- Numerical errors are the deviation from the analytical derived speed and the initial shape of the solitary wave

Solitary Wave Benchmark





Compaction of the matrix - updating the porosity

REMEMBER:

$$\frac{D\phi}{Dt} = (1-\phi)\nabla \cdot \boldsymbol{v_s}$$

where

$$\frac{D\phi}{Dt} = \frac{\delta\phi}{\delta t} + \boldsymbol{v_s}\nabla\phi$$

Backward Euler Method

$$\phi_{t+1} = \phi_t + (1-\phi_t)\nabla \cdot v_{s_t}$$

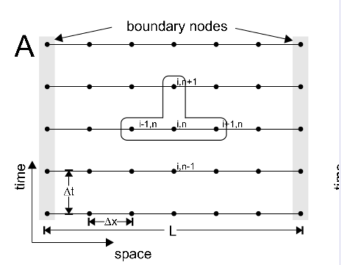
Crank Nicolson Method

$$\phi_{t+1}^{i+1} = \phi_t + \frac{1}{2}\left((1-\phi_t)\nabla \cdot v_{s_t} + (1-\phi_t^i)\nabla \cdot v_{s_t}^i\right)$$

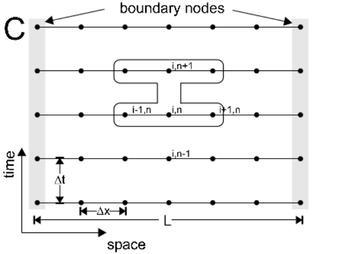
Which method should produce the better results?

Compaction of the matrix - updating the porosity

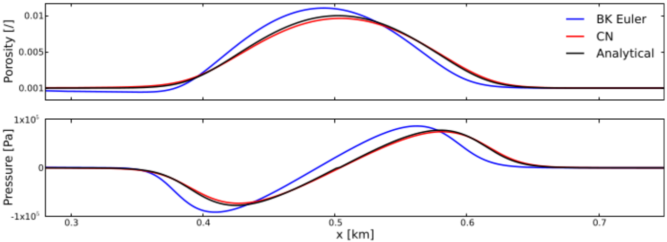
Backward Euler Method



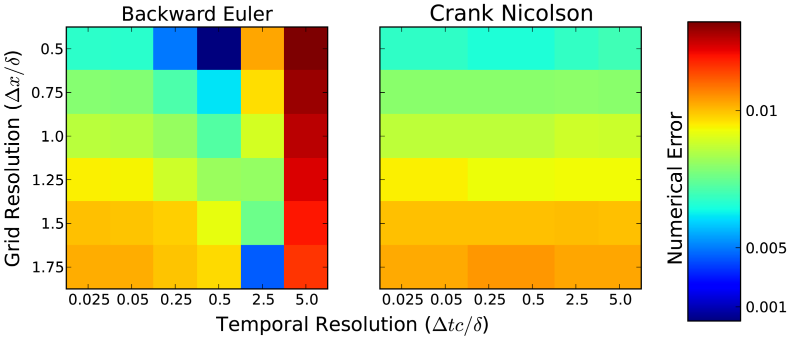
Crank Nicolson Method



Solitary Wave Benchmark



Solitary Wave Benchmark



Δx spatial resolution, Δt time step, δ compaction length, c wave speed

Describe the results! Which method would you use (for what?) ?

Example I - Melt migration in rift settings

avi

Example I - Melt migration in rift settings

What will happen if we increase the viscosity of solid matrix or the tensile strength?

Example I - Melt migration in rift settings

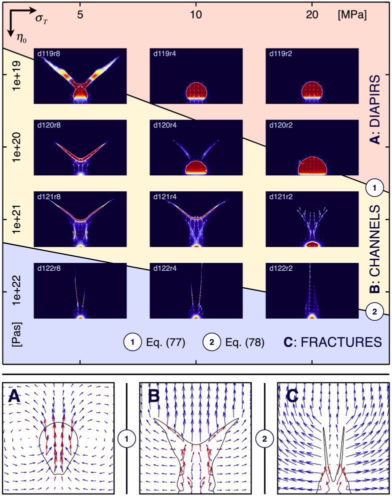
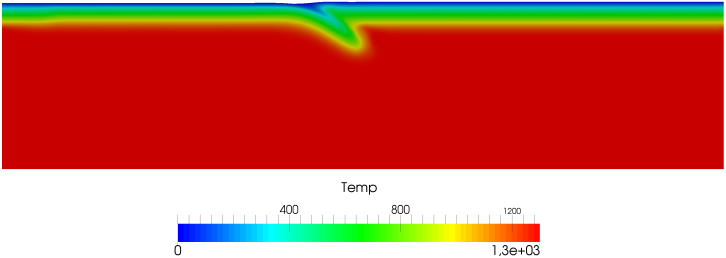
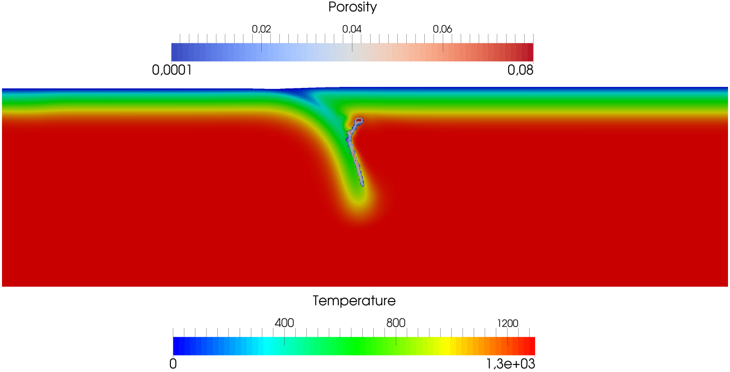


Figure: Keller et al., 2013

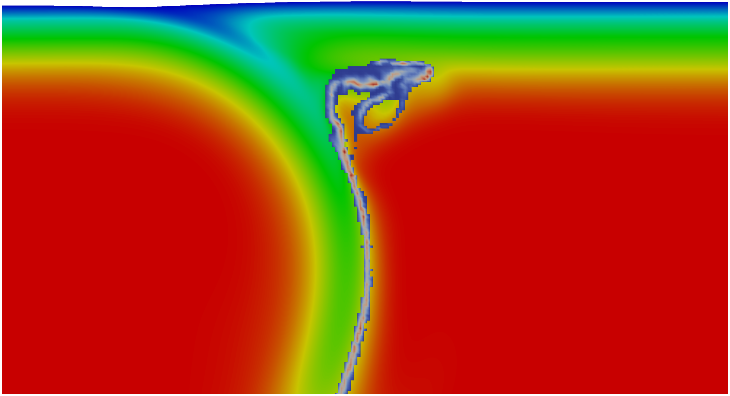
Example II - Modeling fluid flow in subduction zones



Example II - Modeling fluid flow in subduction zones



Example II - Modeling fluid flow in subduction zones



MATLAB Exercise

- Get familiar with the code! What kind of numerical techniques does the code use?
- What are the boundary conditions for velocity? Try to study the code to learn about that them! What impact do they have?
- Set the following parameter: $K0 = 5 \cdot 10^{-15} km^2, \rho_s = 3 \frac{g}{cm^3}, \eta_s = 1e20 Pa \cdot s, xsize = 20km, ynum = 80, \rho_f = 2.5 \frac{g}{cm^3}$. Check the units!
- Run the code! What do you observe? How do permeability (change to 5e-11 m^2) or fluid density (change to 2900 $\frac{kg}{m^3}$) influence the results? How does changing the resolutions change the results?

MATLAB Exercise

- Get familiar with the code! What kind of numerical techniques does the code use?
- What are the boundary conditions for velocity? Try to study the code to learn about that them! What impact do they have?
- Set the following parameter: $K0 = 5 \cdot 10^{-15} km^2, \rho_s = 3 \frac{g}{cm^3}, \eta_s = 1e20 Pa \cdot s, xsize = 20 km, ynum = 80, \rho_f = 2.5 \frac{g}{cm^3}$. Check the units!
- Run the code! What do you observe? How do permeability (change to 5e-11 m^2) or fluid density (change to 2900 $\frac{kg}{m^3}$) influence the results? How does changing the resolutions change the results?

MATLAB Exercise

- Get familiar with the code! What kind of numerical techniques does the code use?
- What are the boundary conditions for velocity? Try to study the code to learn about that them! What impact do they have?
- Set the following parameter: $K0 = 5 \cdot 10^{-15} km^2, \rho_s = 3 \frac{g}{cm^3}, \eta_s = 1e20 Pa \cdot s, xsize = 20 km, ynum = 80, \rho_f = 2.5 \frac{g}{cm^3}$. Check the units!
- Run the code! What do you observe? How do permeability (change to 5e-11 m^2) or fluid density (change to 2900 $\frac{kg}{m^3}$) influence the results? How does changing the resolutions change the results?

MATLAB Exercise

- Get familiar with the code! What kind of numerical techniques does the code use?
- What are the boundary conditions for velocity? Try to study the code to learn about that them! What impact do they have?
- Set the following parameter: $K0 = 5 \cdot 10^{-15} km^2, \rho_s = 3 \frac{g}{cm^3}, \eta_s = 1e20 Pa \cdot s, xsize = 20km, ynum = 80, \rho_f = 2.5 \frac{g}{cm^3}$. Check the units!
- Run the code! What do you observe? How do permeability (change to 5e-11 m^2) or fluid density (change to 2900 $\frac{kg}{m^3}$) influence the results? How does changing the resolutions change the results?

MATLAB Exercise

- Get familiar with the code! What kind of numerical techniques does the code use?
- What are the boundary conditions for velocity? Try to study the code to learn about that them! What impact do they have?
- Set the following parameter: $K0 = 5 \cdot 10^{-15} km^2, \rho_s = 3\frac{g}{cm^3}, \eta_s = 1e20 Pa \cdot s, xsize = 20km, ynum = 80, \rho_f = 2.5\frac{g}{cm^3}$. Check the units!
- Run the code! What do you observe? How do permeability (change to 5e-11 m^2) or fluid density (change to 2900 $\frac{kg}{m^3}$) influence the results? How does changing the resolutions change the results?

MATLAB Exercise